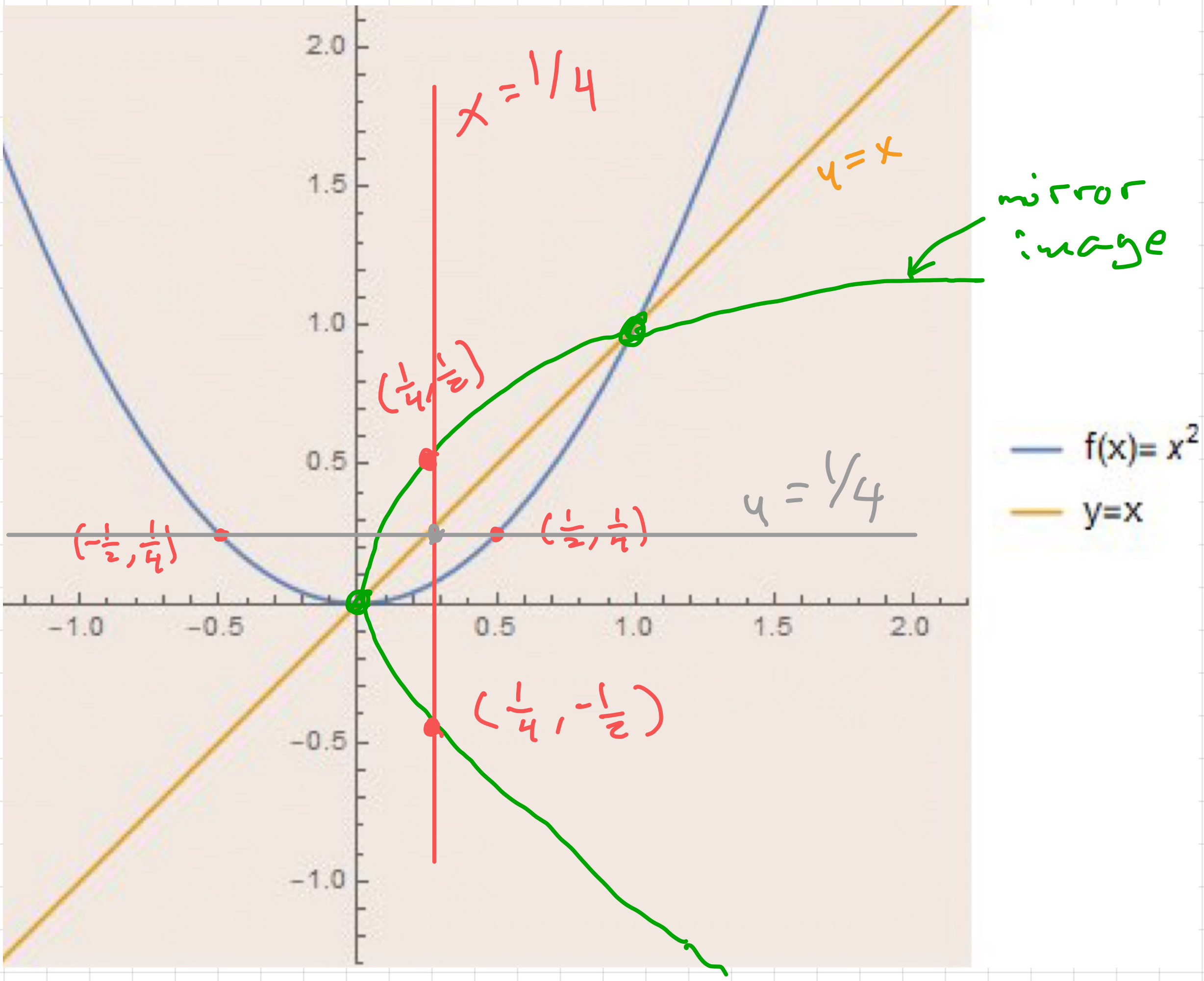


"aspect ratio" = 1

(1) Draw a picture of the mirror image of $y = x^2$ across the line $y = x$.

(2) What are the coordinates of the mirror image of $(\frac{1}{2}, \frac{1}{4})$?

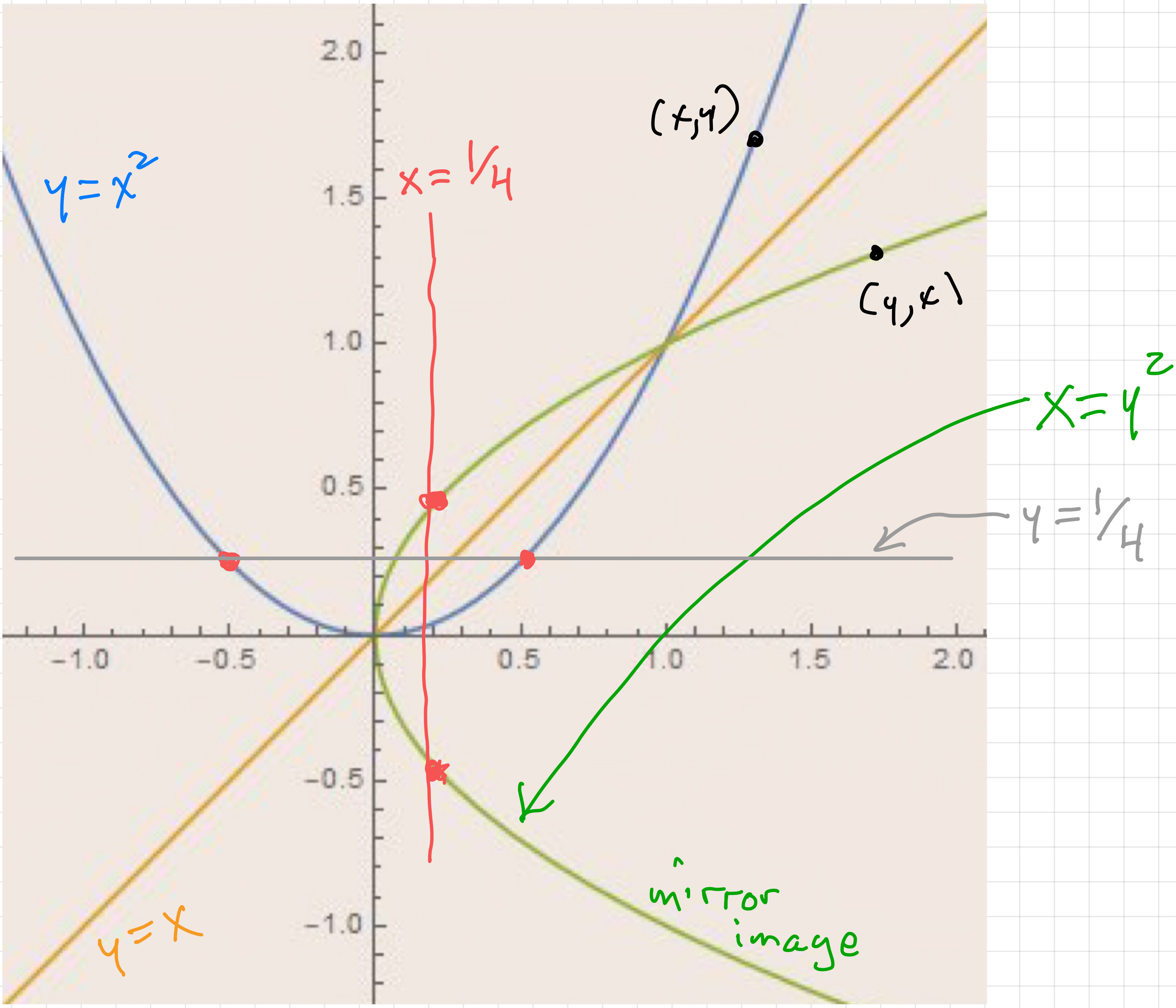
$(1/4, 1/2)$

(3) What are the coordinates of the mirror image of $(-\frac{1}{2}, \frac{1}{4})$?

$(1/4, -1/2)$

(4) Where does the vertical line $x = \frac{1}{4}$ intersect the mirror image of the parabola?

(5) Is the mirror image of the parabola the graph of a function? No! Doesn't satisfy VLP.



notes:

- If a point (x,y) is on $y=x^2$ then (y,x) is on its mirror image across $y=x$. So an equation for the mirror image curve is $x=y^2$, obtained from $y=x^2$ by interchanging x and y.
- The vertical line $x=1/4$ is the mirror image of the horizontal line $y=1/4$ across $y=x$. So the mirror image curve $x=y^2$ fails to satisfy VLP because the original parabola $y=x^2$ fails to satisfy HLP.

As we move into Chapter 6 of Stewart's book note that we will be covering the starred section 6.2*, 6.3*, 6.4* which describe logarithm functions first, before exponential functions. This is indicated at the course web site:

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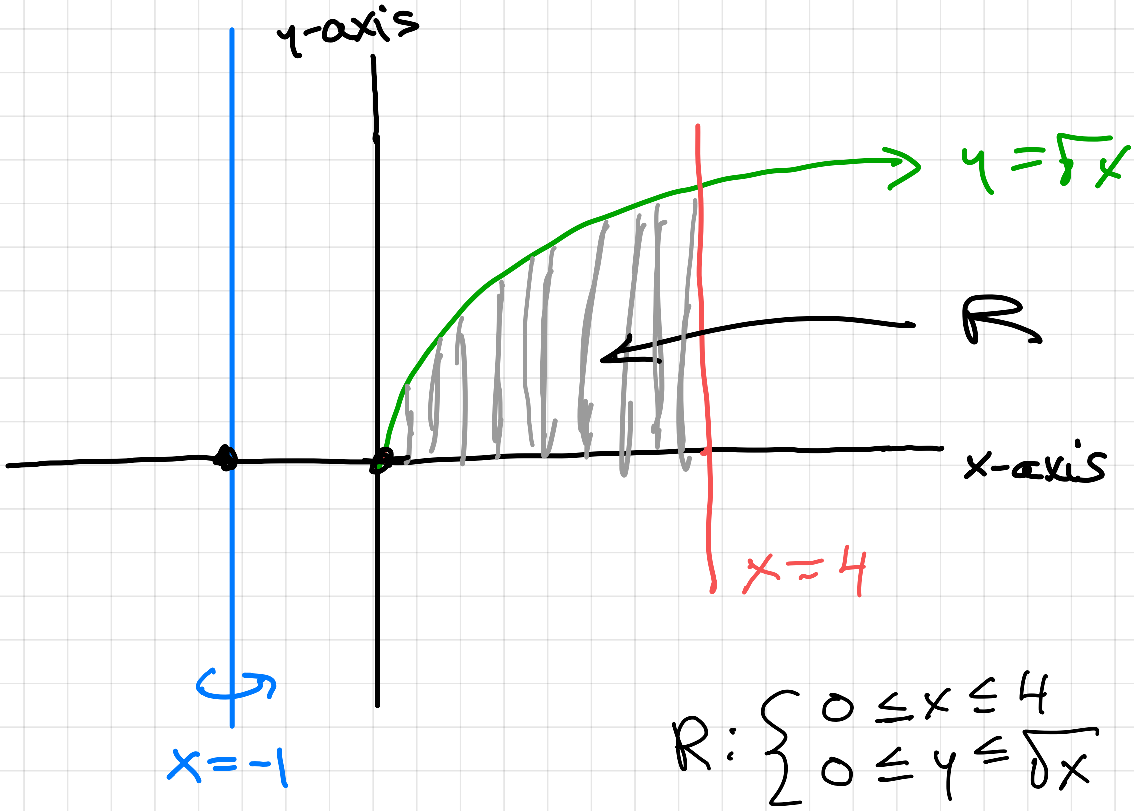
Calculus II
Math 2423, Section 010
Spring Semester 2021

In this course much of the material in Chapters 4 through 8 of the textbook ("Calculus (8th edition)" by James Stewart) will be covered. The list below shows the specific sections that will be studied, and indicates others that will only be discussed briefly in class. For each of the sections to be studied carefully, you will find a list of appropriate exercises to assist in mastering the concepts. (There are a large number of problems listed but many of them are quite repetitive.)

SECTION	PAGE	PROBLEMS
Chapter 1 Review	96	1-2, 5-8
Chapter 2 Review	196	13-44, 47-48, 53-54
Chapter 3 Review	286	1-6, 17-32, 36, 40, 53-60
4.1	303	17-18
4.2	316	5-6, 33-53, 59-68
4.3	327	7-59
4.4	336	1-42, 45-46, 55-58
4.5	346	1-52, 59-60
Chapter 4 Review	349	3, 7-31, 35
5.1	362	1-40
5.2	374	1-30, 39-42, 47-61
5.3	381	1-20, 29-32, 37-43
5.5	391	1-10, 13-15
Chapter 5 Review	393	1-17, 19-24, 31
6.1	406	1-44
6.2*	445	1-58, 61-74, 76-78, 87
6.3*	449	1-62, 67-75, 83-94, 96-98, 105
6.4*	463	4-18, 25-52
6.6	481	1-47, 51-54, 57-70
6.8	499	1-4, 7-68, 75-82
Chapter 6 Review	505	1-59, 63-85, 92-117
7.1	516	1-57, 65-66
7.2	524	1-50, 55-58, 61-64
7.3	526	1-30, 33-34
7.4	541	1-32, 39-48
7.5	547	1-80
7.8	574	1-2, 5-42, 57-59
Chapter 7 Review	577	1-50, 55-60, 72-76
8.1	588	1-2, 9-20, 35, 39

example 3

rotate R around the line $x=-1$ to get solid S_3



$$R: \begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$$

shell method:

$$\text{Volume}(S_3) = \int_0^4 2\pi(x+1)\sqrt{x}\,dx = \int_0^4 2\pi x^{3/2} + 2\pi x^{1/2}\,dx$$

$$= \frac{544}{15}\pi$$

see next pages

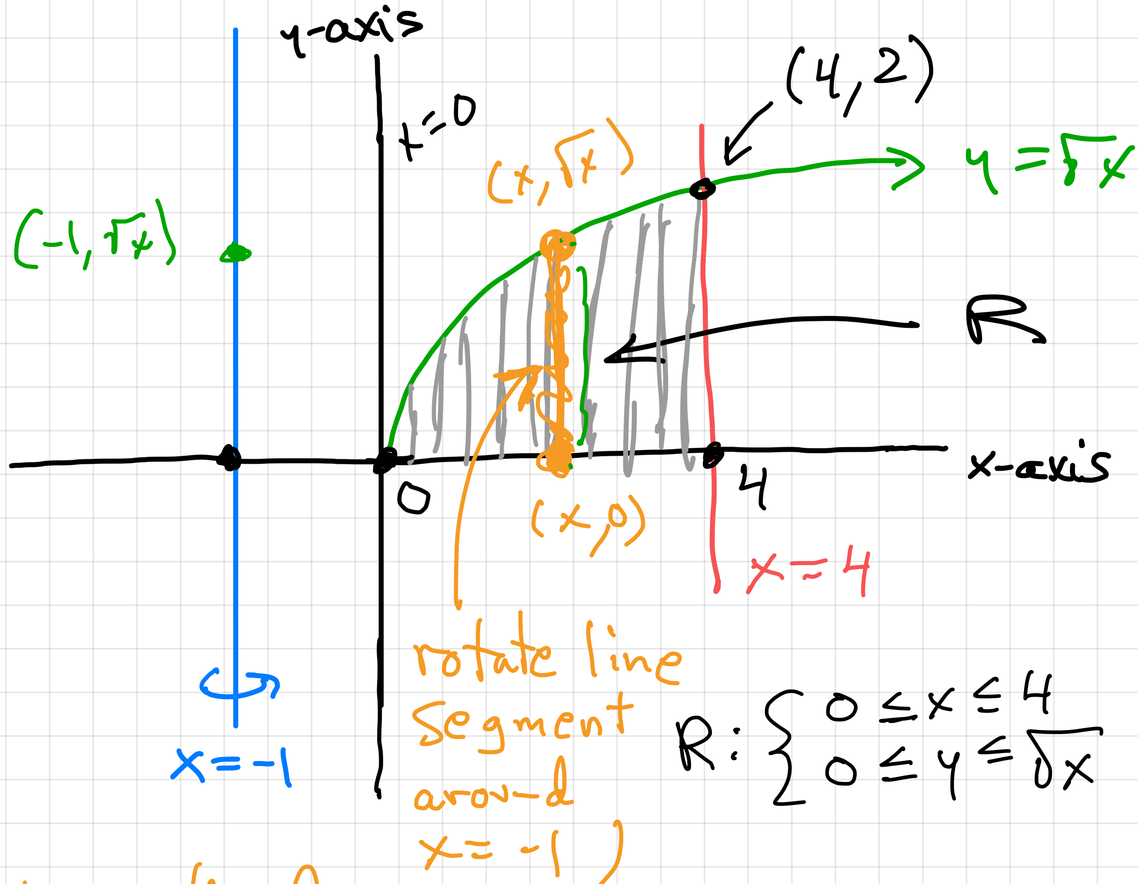
washer method:

$$\text{Volume}(S_3) = \int_0^2 \pi 5^2 - \pi(y^2+1)^2\,dy$$

$$= \int_0^2 24\pi - 2\pi y^2 - \pi y^4\,dy = \frac{544}{15}\pi$$

Volume example 3

rotate R around $x=-1$



$$R: \begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$$

shell method: take reference line x-axis.

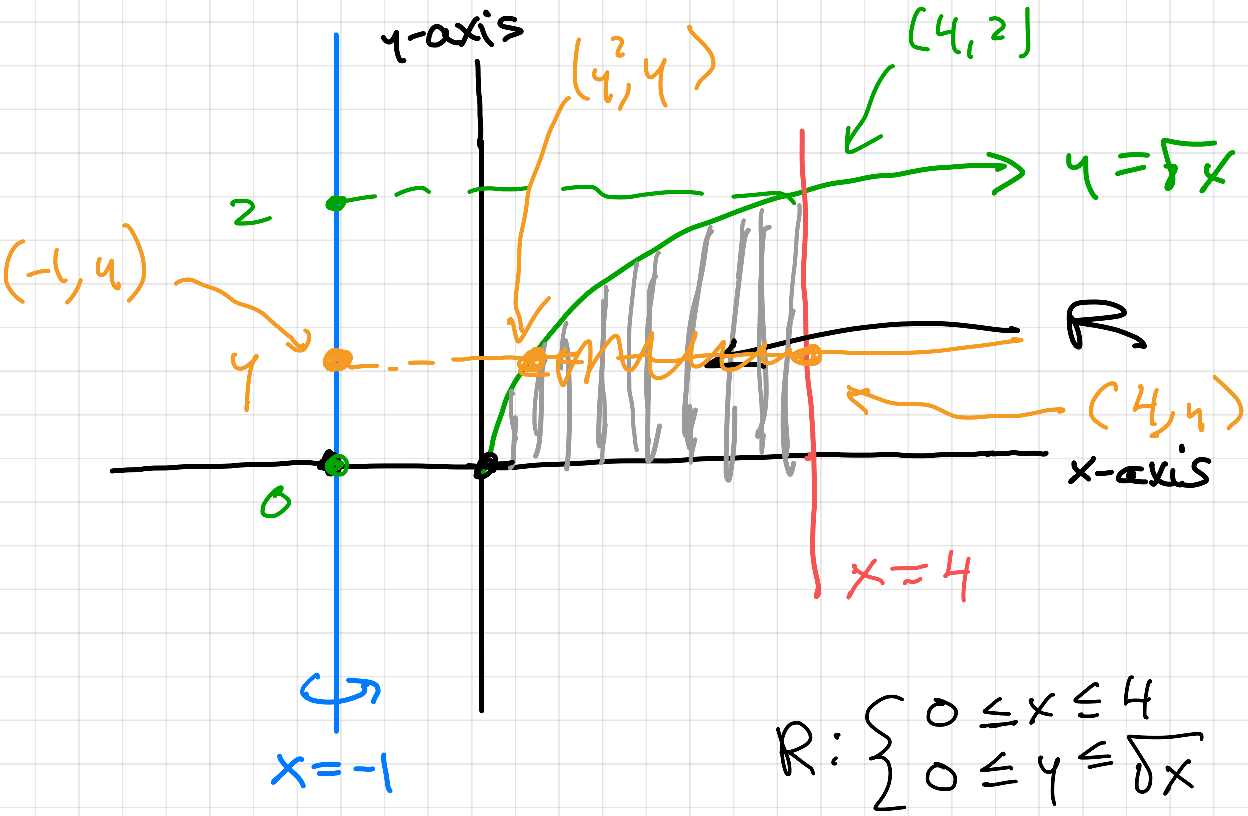
$r = \text{radius} = x-(-1) = x+1$

area of cylinder $= 2\pi(x+1)\sqrt{x}$

$w = \sqrt{x}$

$$\text{Volume} = \int_0^4 2\pi(x+1)\sqrt{x}\,dx$$

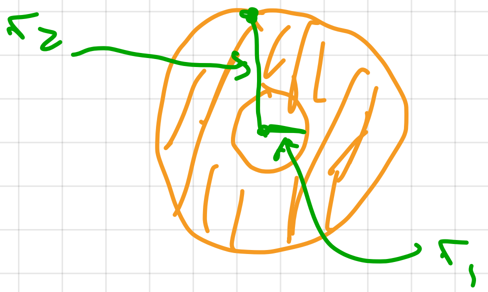
$$= 2\pi\int_0^4 x^{3/2} + x^{1/2}\,dx = \frac{544}{15}\pi$$



$$R: \begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$$

Disk method: reference line $x=-1$

rotate interval around $x=-1$



washer

$r_2 = 5$

$r_1 = y^2 - (-1)$
$= y^2 + 1$

area(washer)
$= \pi 5^2 - \pi (y^2+1)^2$

$$\text{Volume} = \int_0^2 \text{area(washer)}\, dy$$

$$= \int_0^2 \pi \left(25 - (y^2+1)^2\right) dy = \frac{544}{15}\pi$$

Inverse Functions

Only "some", not "all"!

Some functions $f(x)$ are linked with an "inverse function" $f^{-1}(x)$, and often the inverse function has interesting properties.

example from last class:

The function $f(x) = \frac{x-2}{3x+1}$ has inverse function

$f^{-1}(x) = \frac{x+2}{1-3x}$. Let's calculate $f^{-1}(f(x))$.

$f(x)$

$$f^{-1}(f(x)) = \frac{f(x)+2}{1-3f(x)} = \frac{\frac{x-2}{3x+1}+2}{1-3\,\frac{x-2}{3x+1}} \cdot \frac{3x+1}{3x+1}$$

$$= \frac{(x-2)+2(3x+1)}{(3x+1)-3(x-2)} = \frac{x-\cancel{2}+6x+\cancel{2}}{\cancel{3x}+1-\cancel{3x}+6}$$

$$= \frac{7x}{7} = x$$

- So $f^{-1}(f(x)) = x$ which in essence says that the function f^{-1} undoes the function f. This is the idea of "inverse".
- It can also be checked that $f(f^{-1}(x)) = x$.

example $f(x) = 3x + 7$

Try to solve the equation $y = f(x)$ for x.

$$y = 3x + 7 \quad = f(x)$$

subtract 7

$$3x = y - 7$$

divide by 3

$$x = \frac{1}{3}y - \frac{7}{3} \quad = f^{-1}(y)$$

Now observe that:

$f^{-1}(y)$ is the number x for which $f(x) = y$.

E.G.

$f(0) = 7 \Rightarrow f^{-1}(7) = 0$

$f(3) = 16 \Rightarrow f^{-1}(16) = 3$

$f(-7/3) = 0 \Rightarrow f^{-1}(0) = -7/3$

So, if $f(x)$ is to have an inverse function then we need to know that for each number y there is at most one number x with $f(x) = y$.

A function $f(x)$ that satisfies this property is called a one-to-one function.

Thus each one-to-one function has an inverse function (and vice-versa).

On pages 400–401, Stewart writes:

2 Definition Let f be a one-to-one function with domain A and range B. Then its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \iff f(x) = y$$

for any y in B.

and

1 Definition A function f is called a **one-to-one function** if it never takes on the same value twice; that is,

$$f(x_1) \neq f(x_2) \qquad \text{whenever } x_1 \neq x_2$$

<u>example</u> $f(x) = x^2$ is <u>not</u> one-to-one because $f(\frac{1}{2}) = \frac{1}{4}$ and $f(-\frac{1}{2}) = \frac{1}{4}$ (but $\frac{1}{2} \neq -\frac{1}{2}$).