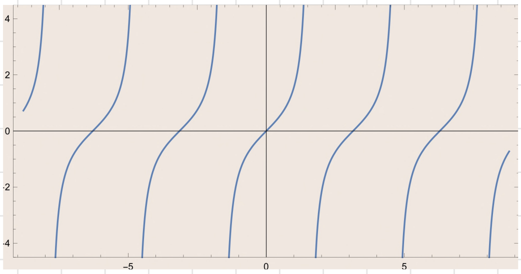
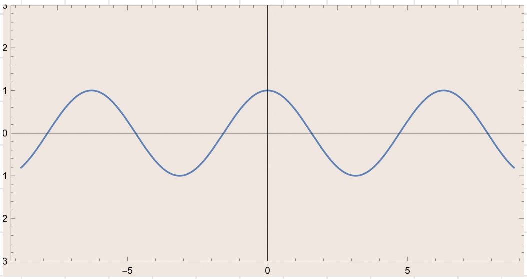
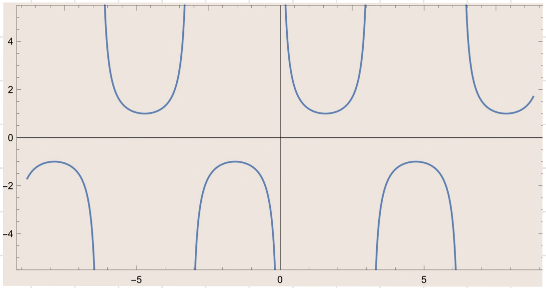
(a)



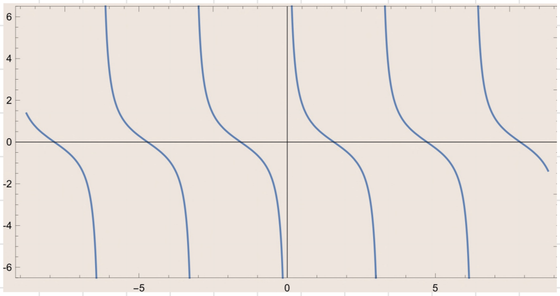
(b)



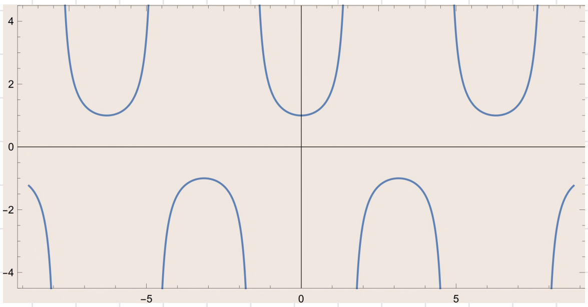
(c)



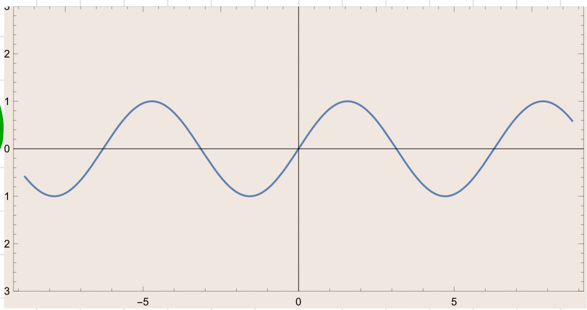
(d)



(e)



(f)



Can you identify the graphs?

$y = \sin x$ f

$y = \cos x$ b

$y = \tan x$ a

$y = \sec x$ e

$y = \cot x$ d

$y = \csc x$ c

Exam 2 – In-Class Component
Math 2423, 3/24/21

Instructions: Your written solutions to problems on this test are to be scanned into a pdf file and submitted via Canvas at the end of the class period. As cover sheet you will need to attach the signed integrity statement which has already been provided for you.

Use your own paper to write up solutions, and clearly label each problem number. Answers should be written in simplified form as appropriate. To get full credit on your work you must clearly indicate the thought processes that you have used to solve each problem. Please draw robust sketches with all prominent features clearly labeled for each problem which requires it.

PROBLEM 1. *(25 points)* Let $\mathcal{R}$ be the region in the first quadrant bounded by the curve $y = \ln(x)$ and the lines $y = 0$, $x = 1$ and $x = e$. Let $\mathcal{S}$ be the solid obtained by rotating $\mathcal{R}$ around the x-axis.

(a) Draw a robust picture of the region $\mathcal{R}$ with all curves and the coordinates of any important points clearly labeled.

(b) Express the area of $\mathcal{R}$ as an integral with respect to x.

(c) Express the area of $\mathcal{R}$ as an integral with respect to y.

(d) Express the volume of $\mathcal{S}$ as an integral using the washer method.

(e) Express the volume of $\mathcal{S}$ as an integral using the shell method.

PLEASE NOTE: Do NOT actually calculate any of the integrals in this problem.

PROBLEM 2. *(25 points)* Let $f(x) = x^2 e^x$.

(a) Find all of the critical numbers for $f(x)$.

(b) Determine the intervals on which $f(x)$ is increasing and decreasing.

(c) Give the coordinates for any points in the plane where $f(x)$ has a local extreme point and indicate which are maximums and which are minimums.

(d) Does $f(x)$ have an inverse function? To receive credit you must write a sentence or two which explains your answer.

PROBLEM 3. *(25 points)* Determine the integrals

(a) $\displaystyle\int e^{2x+1}\,dx$

(b) $\displaystyle\int \frac{1}{2x+1}\,dx$

(c) $\displaystyle\int \frac{(\ln(x))^3}{x}\,dx$

(d) $\displaystyle\int_0^{\pi/2} \cos(x)\exp(\sin(x))\,dx$

PROBLEM 4. *(25 points)* Find the derivative $f'(x)$ for each function $f(x)$:

(a) $f(x) = e^{2x+1}$

(b) $f(x) = \dfrac{1}{2x+1}$

(c) $f(x) = \sin(x)\ln(x)$

(d) $f(x) = x^{\sin(x)}$

From Exam 2:

Problem 1. *(25 points)* Let $\mathcal{R}$ be the region in the first quadrant bounded by the curve $y = \ln(x)$ and the lines $y = 0$, $x = 1$ and $x = e$. Let $\mathcal{S}$ be the solid obtained by rotating $\mathcal{R}$ around the x-axis.

(a) Draw a robust picture of the region $\mathcal{R}$ with all curves and the coordinates of any important points clearly labeled.

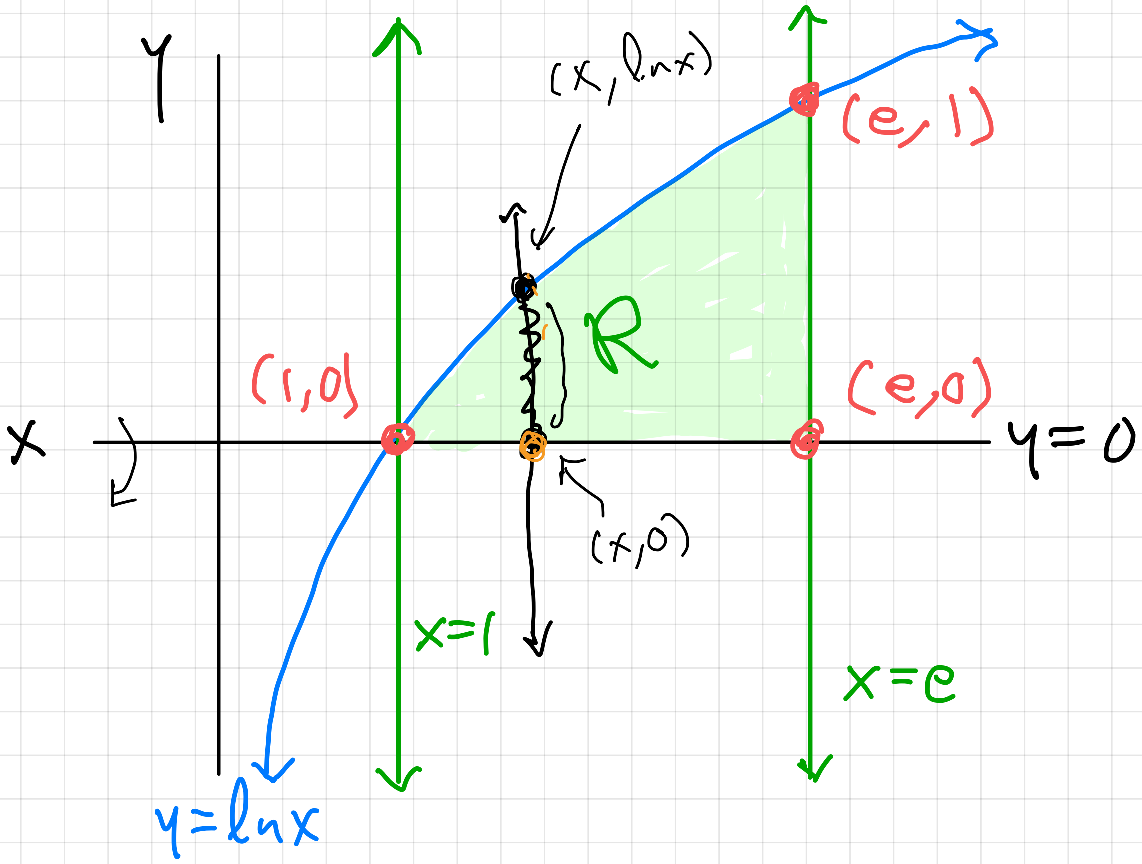
(b) Express the area of $\mathcal{R}$ as an integral with respect to x.

(c) Express the area of $\mathcal{R}$ as an integral with respect to y.

(d) Express the volume of $\mathcal{S}$ as an integral using the washer method.

(e) Express the volume of $\mathcal{S}$ as an integral using the shell method.

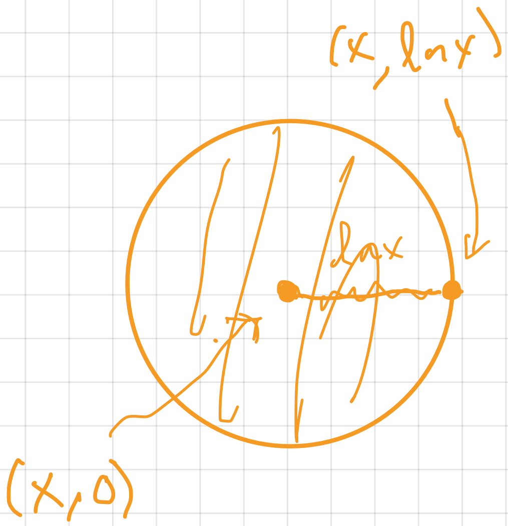
$\ln(e) = 1$

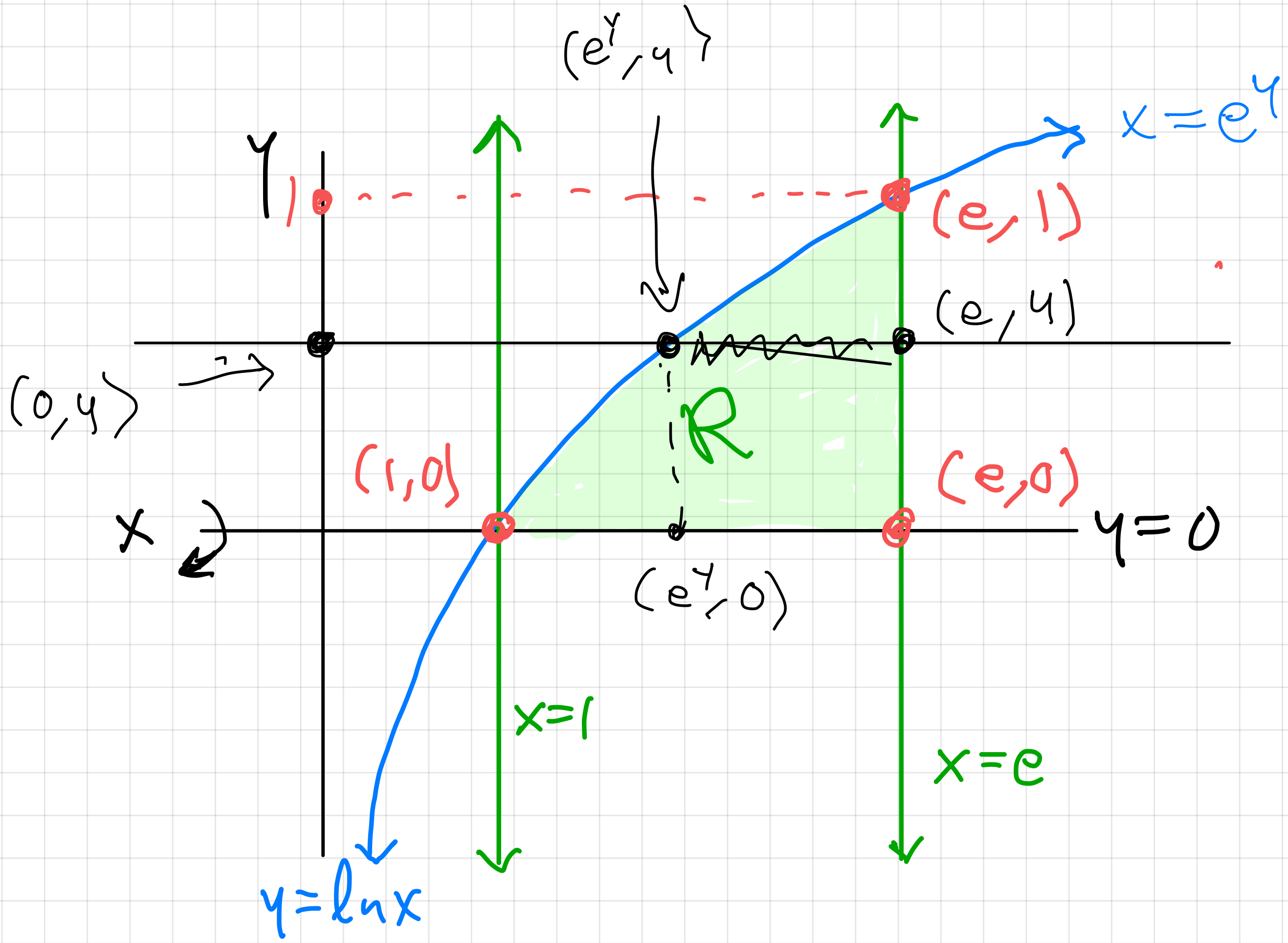


For (b) and (d): The length of the (vertical) slice of R thru $(x, 0)$ is $\ln(x) - 0 = \ln(x)$ for $1 \leq x \leq e$.

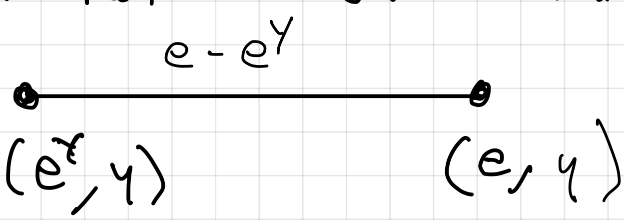
(b) $\text{area}(R) = \int_1^e \ln(x)\, dx$

(c) $\text{Volume}(S) = \int_1^e \pi(\ln(x))^2\, dx$





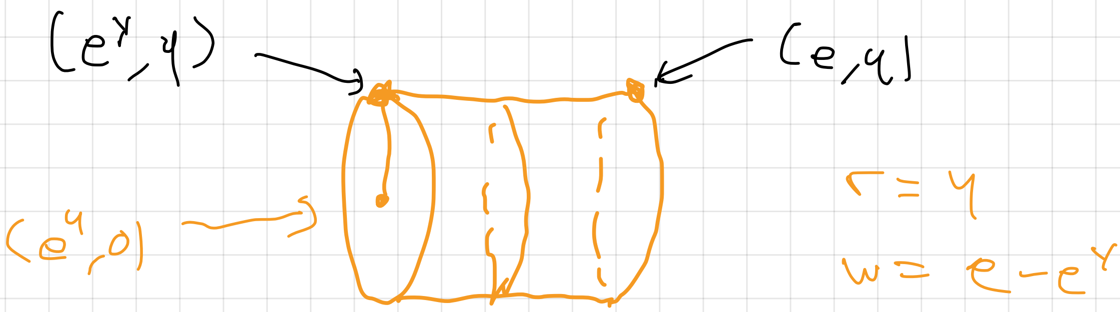
For (c) and (e): The horizontal slice of R thru $(0, y)$ is the segment



for $0 \le y \le 1$.

(c) $\text{Area}(R) = \int_0^1 e - e^y \, dy$

(d) $\text{Volume}(S) = \int_0^1 2\pi y \left(e - e^y\right) dy$



from Exam 2

#4d $\quad f(x) = x^{\sin x}, \quad f'(x) = ?$

Remember (important!)

$$\boxed{b^a = e^{a\ln(b)} = \exp(a\ln b), \text{ for } b>0}$$

$$f(x) = \exp(\sin x \ln x) = e^{\sin(x)\ln(x)}$$

$$f'(x) = \exp(\sin x \ln x) \cdot \frac{d}{dx}\left[\sin x \ln x\right]$$

$$= x^{\sin(x)}\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

Or use logarithmic differentiation

$$y = x^{\sin x}$$

$$\ln(y) = \ln(x^{\sin x}) = \sin(x)\ln(x)$$

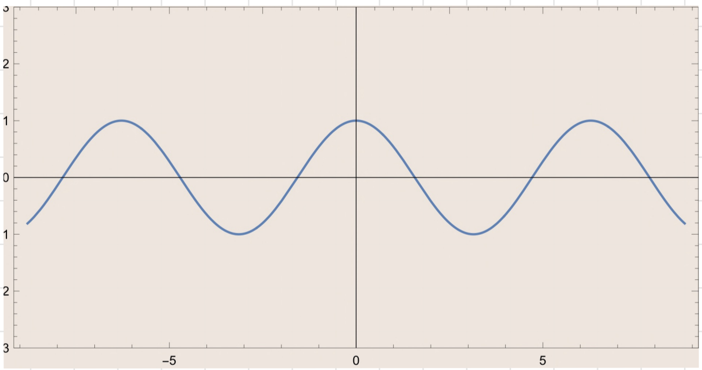
Now take d/dx :

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\ln y) = \frac{d}{dx}(\sin x \ln x)$$

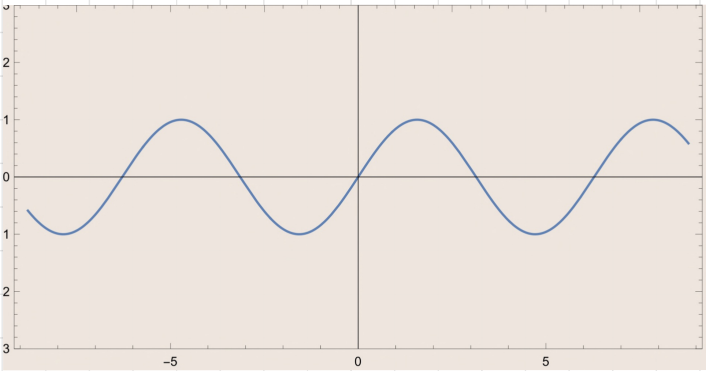
$$= \cos x \ln x + \frac{\sin x}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

None of trig functions satisfy HLP.

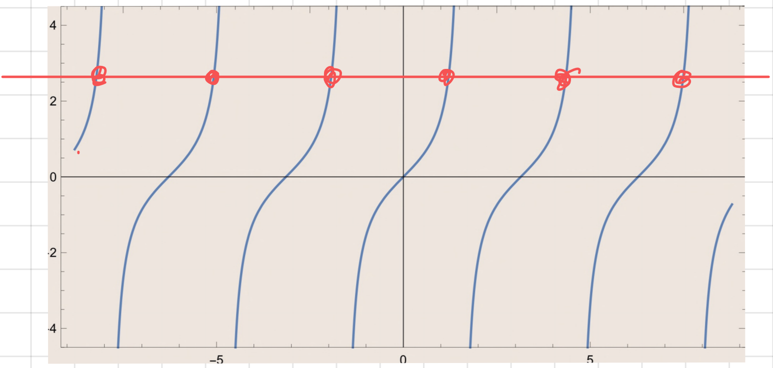


$f(x) = \cos x$

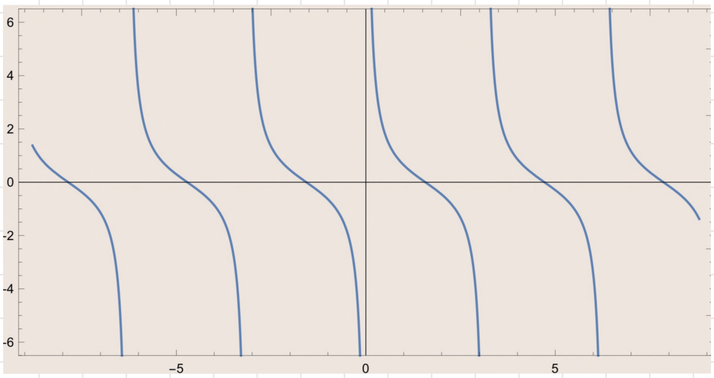


$f(x) = \sin x$

$\cos(x - \pi/2) = \sin x$ period $= 2\pi$

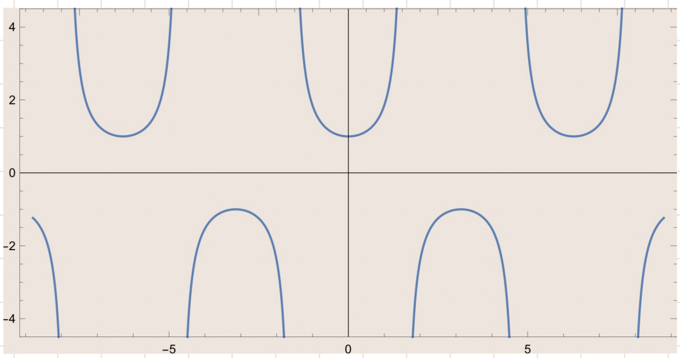


$f(x) = \tan x$

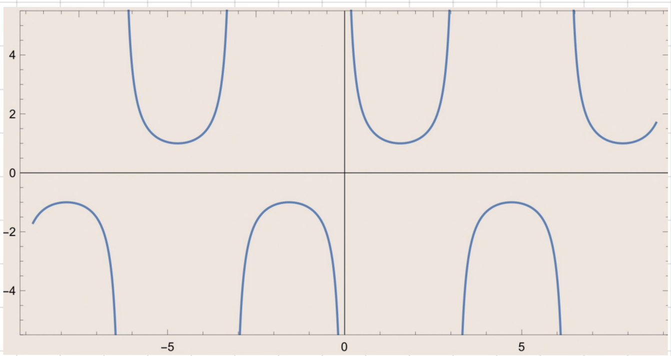


$f(x) = \cot x$

period $= \pi$



$f(x) = \sec x$



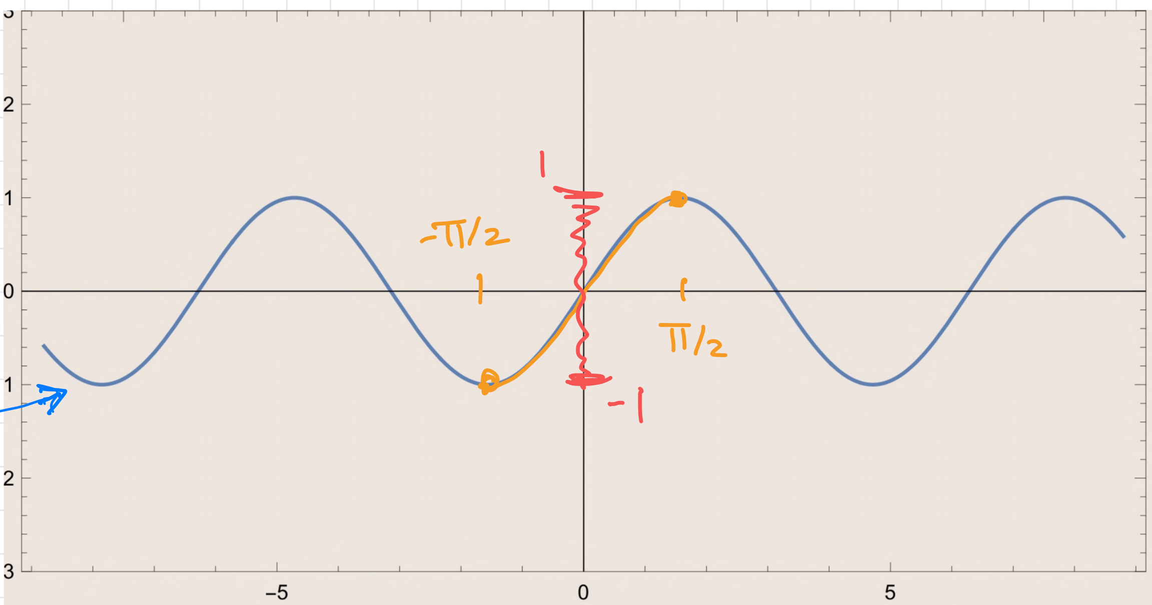
$f(x) = \csc x$

period $= 2\pi$

The inverse sine function

If $y=\sin(x)$ is restricted to the interval $[-\pi/2, \pi/2]$ then it does satisfy HLP.

$Y=\sin(x)$



range$(g) = [-1,1]$

$$g(x) = \sin x, \quad -\pi/2 \le x \le \pi/2$$

The function $g(x)$ is one-to-one, so it has an inverse function. This is the inverse sine function.

$$\sin^{-1}(x) = g^{-1}(x) = \arcsin(x)$$

note that domain$(g) = [-\pi/2, \pi/2]$

range$(g) = [-1, 1]$

In other words: For each number x in $[-1, 1]$

$\sin^{-1}(x) =$ angle θ between $-\pi/2$ and $\pi/2$ for which $\sin(\theta) = x$

$\parallel$

$\arcsin(x)$

Also:

$$\boxed{\begin{aligned} &\bullet\ \sin(\sin^{-1} x) = x \quad \text{for } -1 \le x \le 1 \\ &\bullet\ \sin^{-1}(\sin x) = x \quad \text{for } -\pi/2 \le x \le \pi/2 \end{aligned}}$$

$$\boxed{\begin{aligned} \text{domain}(\sin^{-1}) &= [-1, 1] = \text{range}(g) \\ \text{range}(\sin^{-1}) &= [-\pi/2, \pi/2] = \text{domain}(g) \end{aligned}}$$

Examples:

① $\sin^{-1}(1) =$ angle θ with $\sin\theta = 1$ and $-\pi/2 \le \theta \le \pi/2$

So $\sin^{-1}(1) = \pi/2$.

② $\sin^{-1}(\sqrt{2}/2) = \pi/4$ because $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

③ $\sin^{-1}(-1) \neq 3\pi/2$ even though $\sin(3\pi/2) = -1$

because $3\pi/2$ is not in $[-\pi/2, \pi/2]$.

④ The correct value is $\sin^{-1}(-1) = -\pi/2$.

What does $\frac{d}{dx}\left[\sin^{-1}(x)\right]$ equal?

$$\sin\left(\sin^{-1}(x)\right) = x \quad , \quad -1 \le x \le 1$$

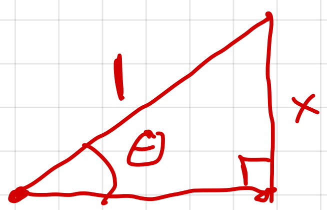
Take $\frac{d}{dx}$ to get:

$$1 = \frac{d}{dx}[x] = \frac{d}{dx}\left[\sin(\sin^{-1}x)\right] = \cos\left(\sin^{-1}(x)\right)\frac{d}{dx}\left[\sin^{-1}x\right]$$

$$\Rightarrow \quad \frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\cos(\sin^{-1}x)} \overset{??}{=} \frac{1}{\sqrt{1-x^2}}$$

Write $\theta = \sin^{-1}(x)$, which means $\sin\theta = x$

If θ is acute then there is a right triangle



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{1}$$

adjacent $= \sqrt{1-x^2}$

Pythagorean Theorem:

adjacent^2 + opposite^2 = hypotenuse^2

So

$$\cos(\sin^{-1}x) = \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \sqrt{1-x^2}$$

Conclude:

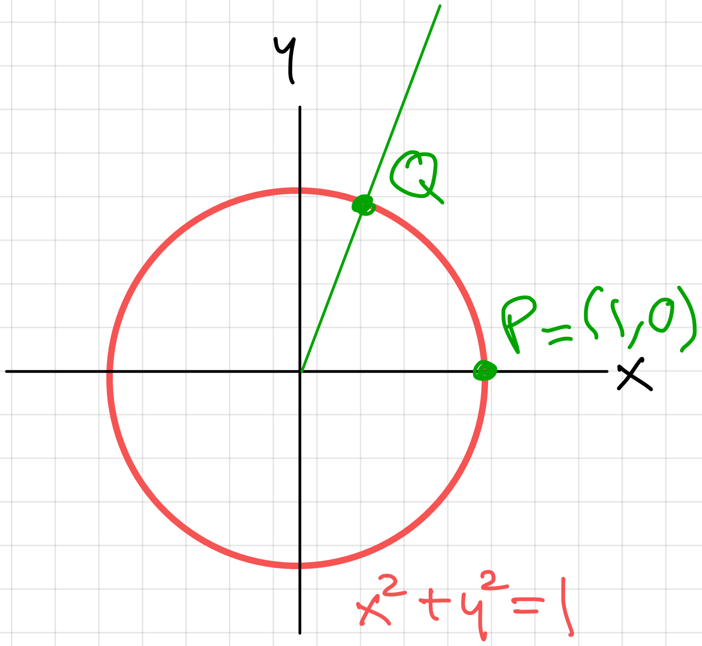
$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$$

(for $-1 \leq x \leq 1$)

Some trig function reminders

If the length of the circular arc from P to Q is θ then Q has coordinates $(\cos\theta, \sin\theta)$.



$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$