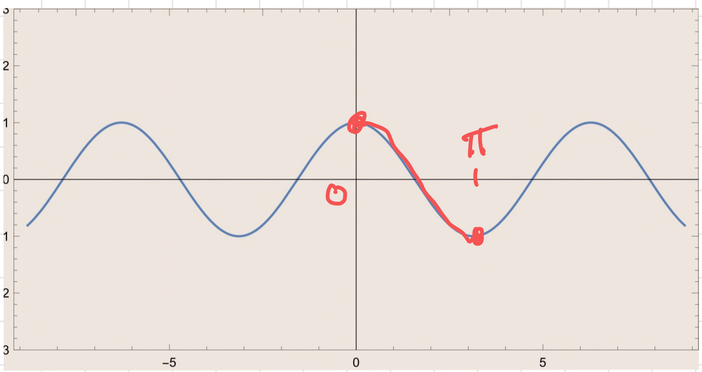
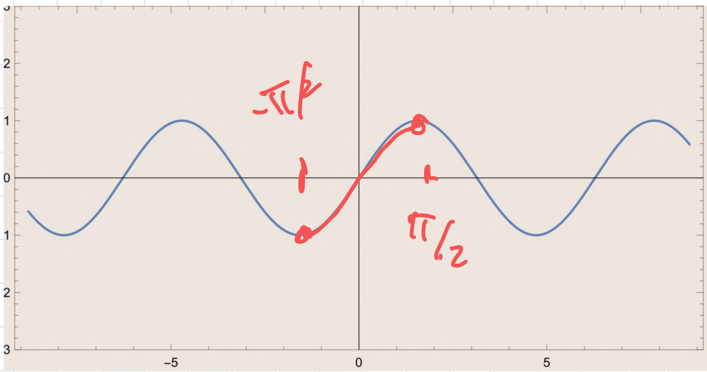
None of trig functions satisfy HLP because of periodicity.

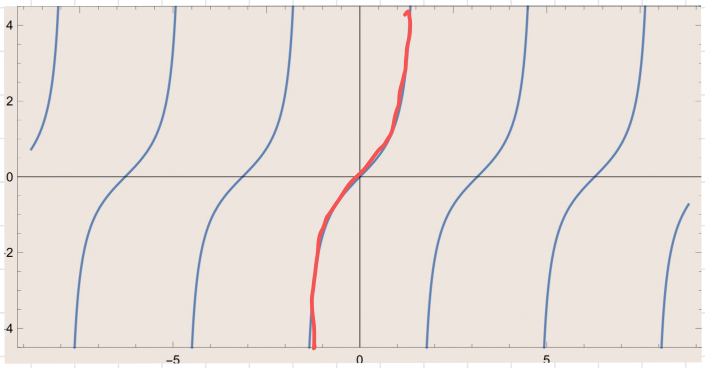


$f(x) = \cos x$

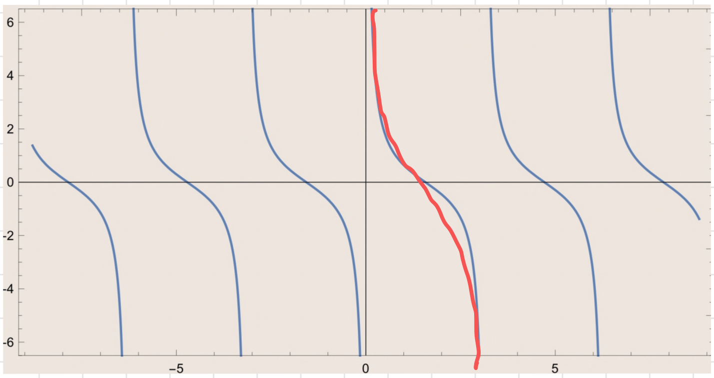


$f(x) = \sin x$

$\cos(x - \pi/2) = \sin x$ period $= 2\pi$

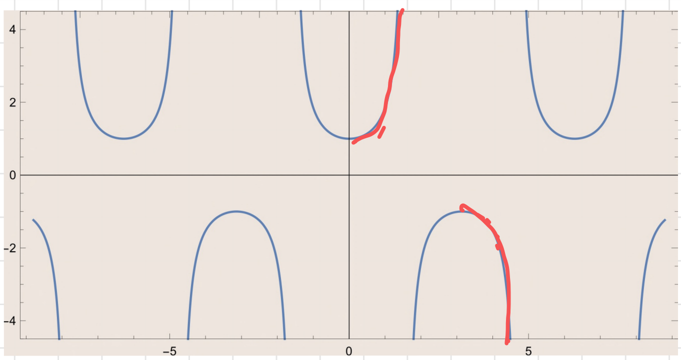


$f(x) = \tan x$

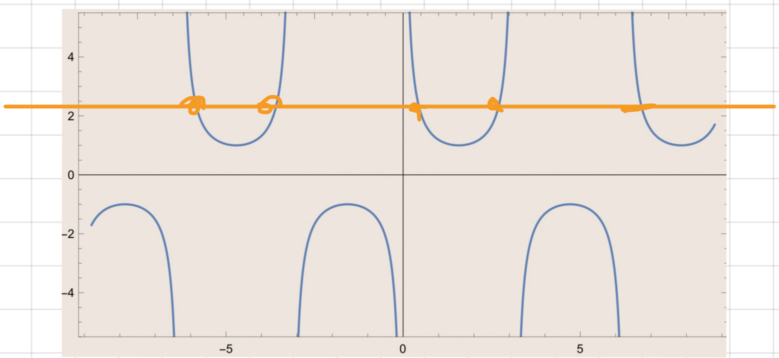


$f(x) = \cot x$

period $= \pi$



$f(x) = \sec x$

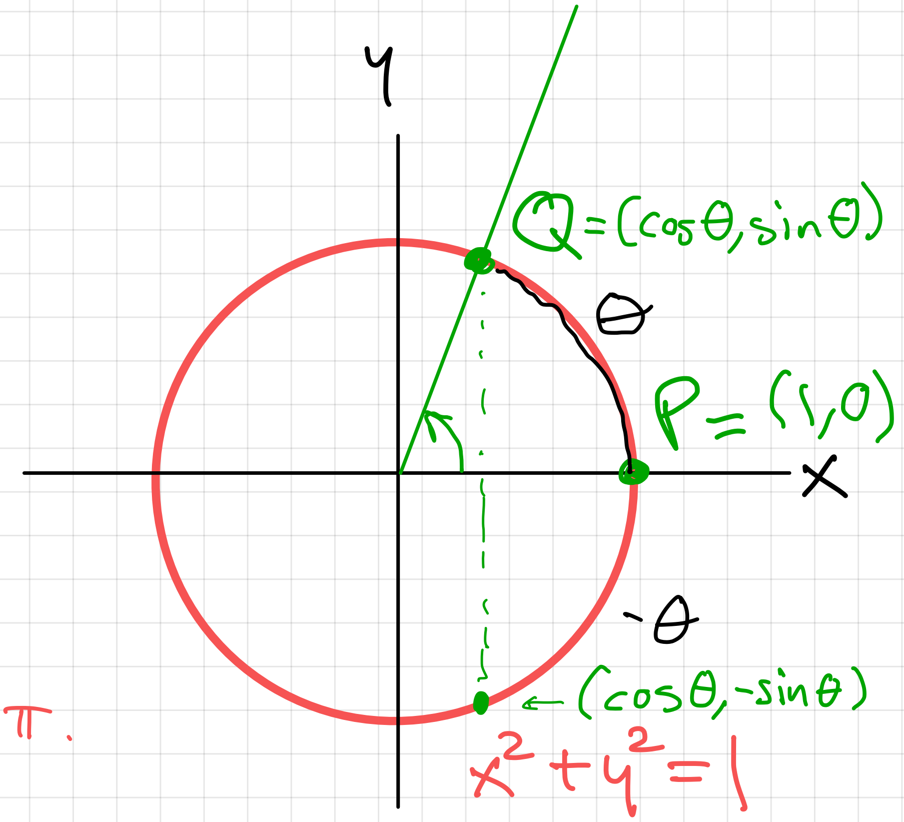


$f(x) = \csc x$

period $= 2\pi$

Some trig function reminders (Quick)

If the length of the circular arc from P to Q is θ then Q has coordinates $(\cos\theta, \sin\theta)$.



unit circle has circumference 2π.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

addition formulas

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

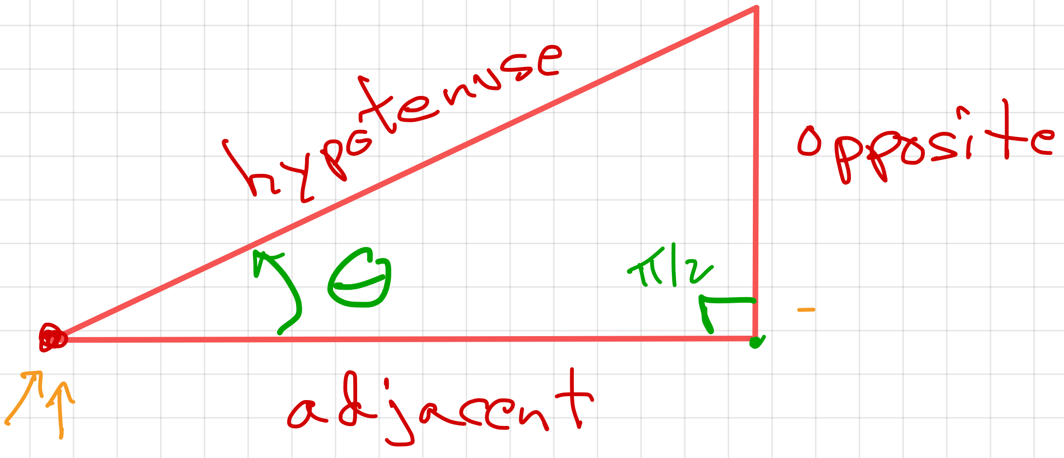
Also:

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

Trig and Right Triangles



Pythagorean Theorem: $adj^2 + opp^2 = hyp^2$

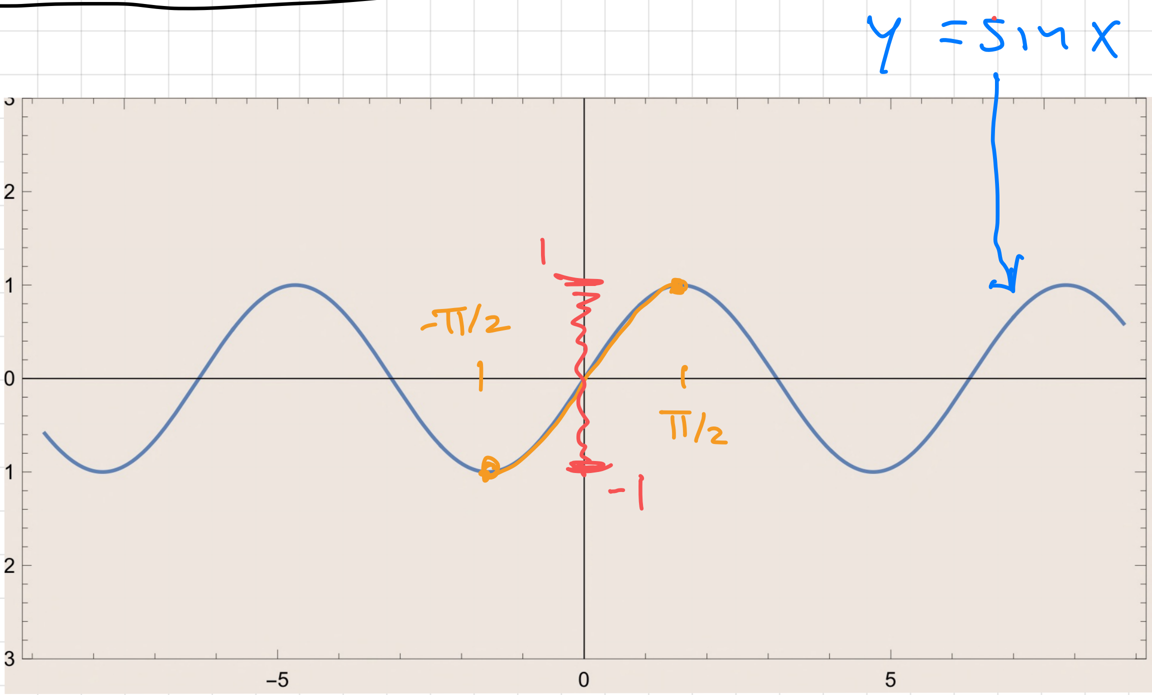
$$\sin\theta = \frac{opp}{hyp}$$

$$\cos\theta = \frac{adj}{hyp}$$

"soh cah toa"

$$\tan\theta = \frac{opp}{adj}$$

The inverse sine function



$$g(x) = \sin x, \quad -\pi/2 \le x \le \pi/2$$

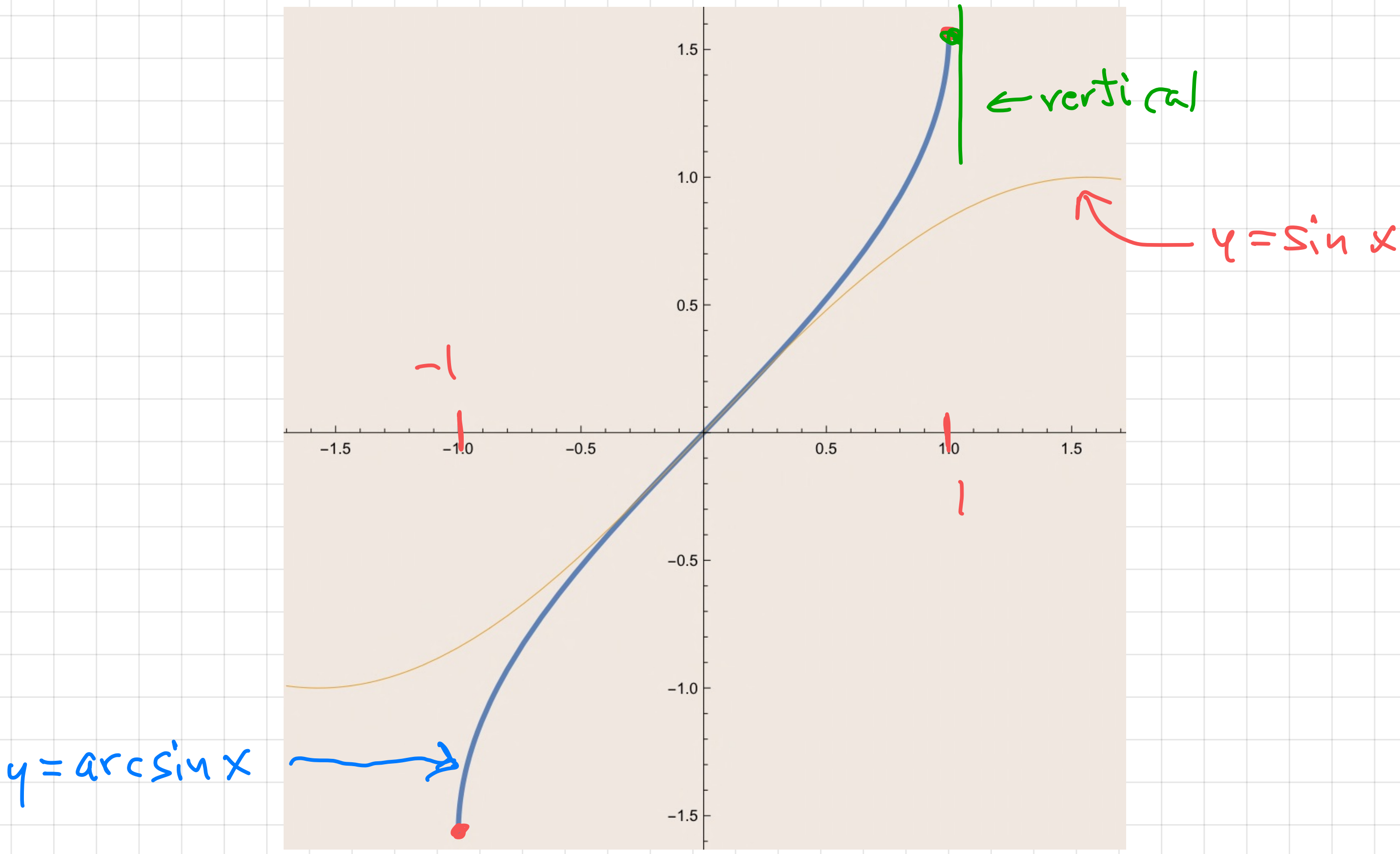
$$\sin^{-1}(x) = \arcsin(x) = g^{-1}(x)$$

$$\boxed{\begin{array}{l} \text{domain}(\sin^{-1}) = [-1, 1] = \text{range}(g) \\ \text{range}(\sin^{-1}) = [-\pi/2, \pi/2] = \text{domain}(g) \end{array}}$$

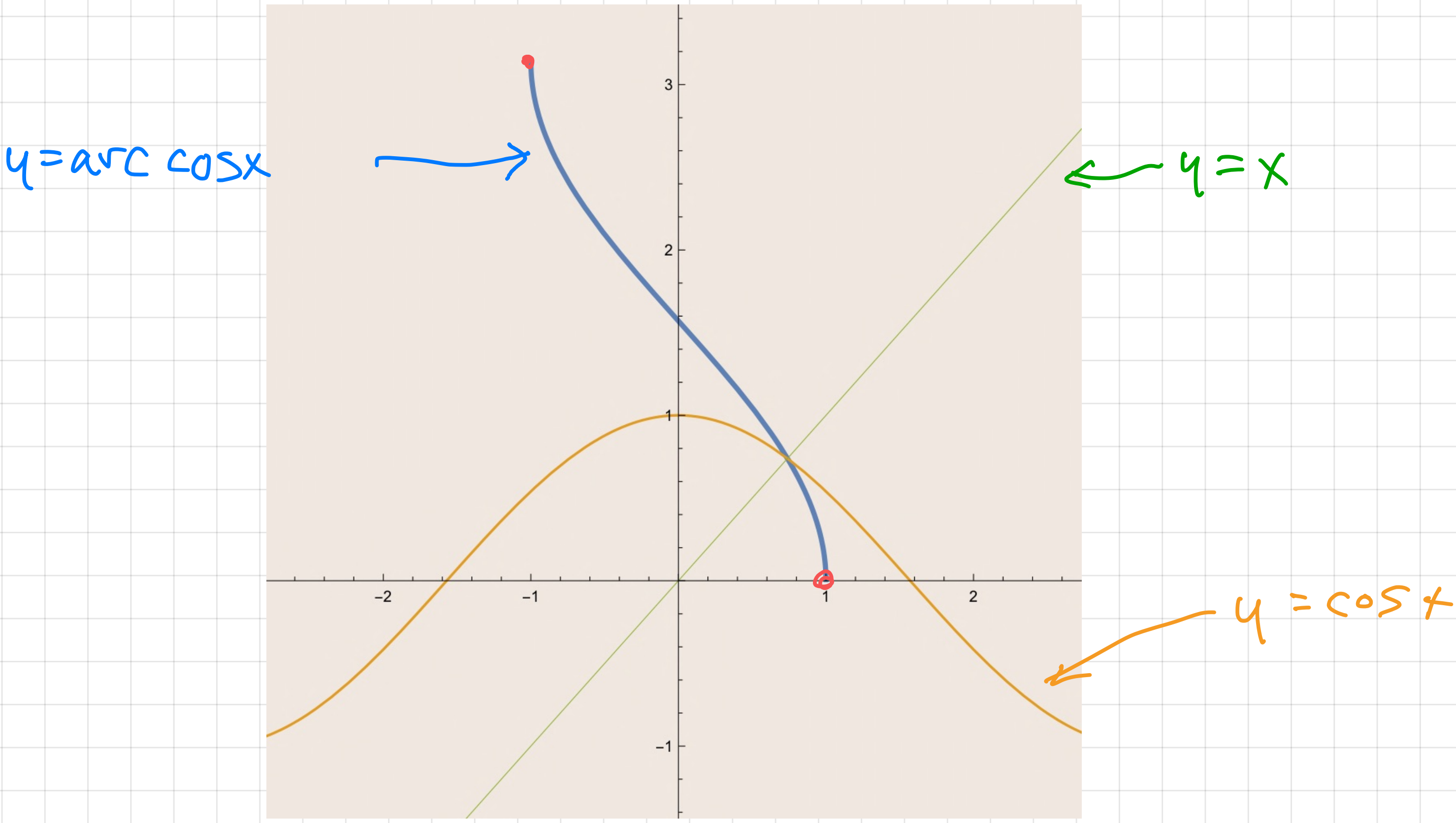
In other words: For each number x in $[-1, 1]$

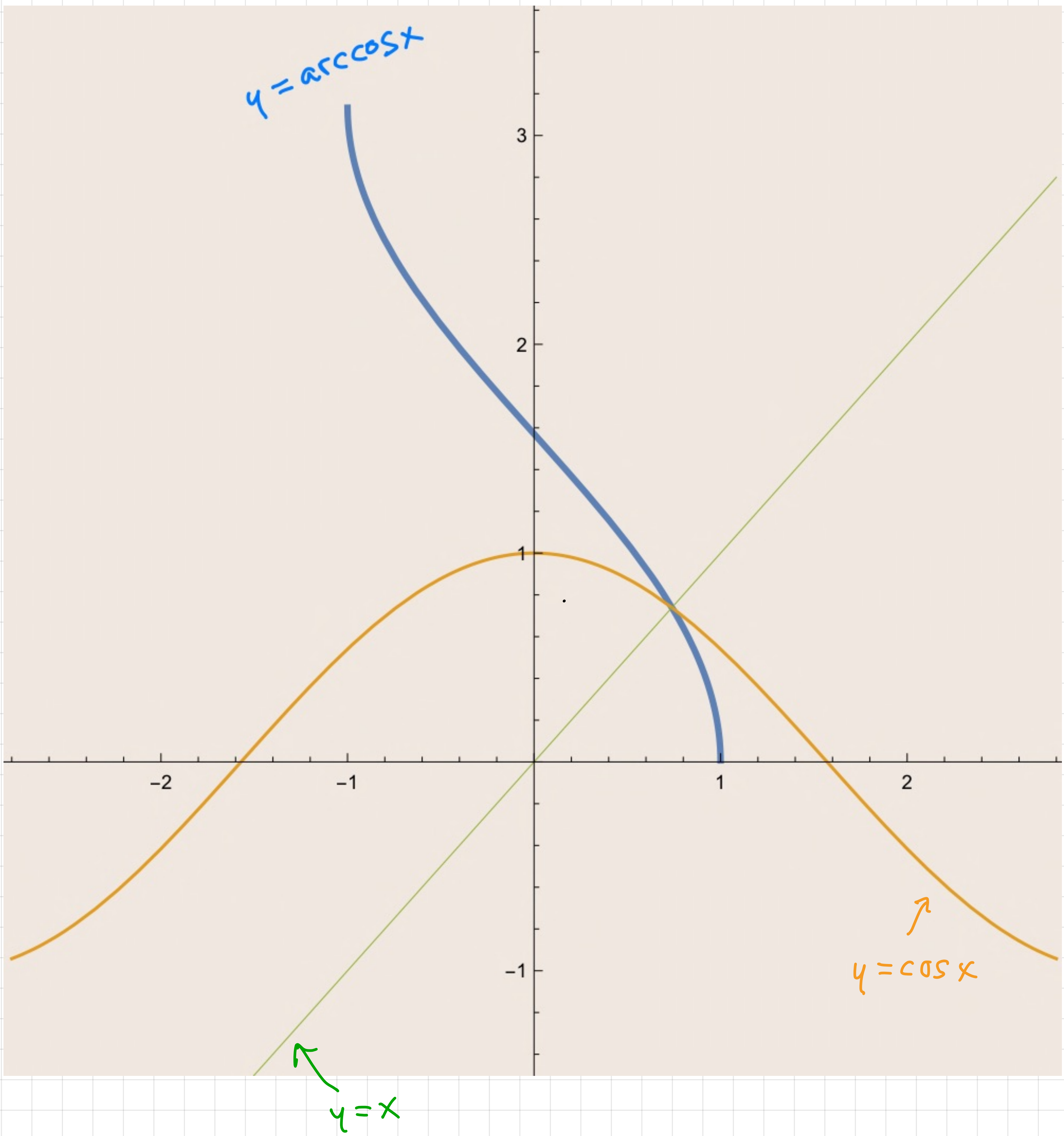
$\sin^{-1}(x) =$ angle θ between $-\pi/2$ and $\pi/2$ for which $\sin(\theta) = x$

- $\sin(\sin^{-1} x) = x$ for $-1 \le x \le 1$
- $\sin^{-1}(\sin x) = x$ for $-\pi/2 \le x \le \pi/2$



For $-1 \le x \le 1$, $\arccos(x)$ is the angle θ between 0 and π with $\cos\theta = x$





Reflect the portion of the graph of $y = \cos x$ across the line $y = x$ to get the graph of $y = \arccos(x)$.

Examples Evaluate / Simplify:

- $\sin^{-1}(-1/2) = -\pi/6$ b/c $\sin(-\pi/6) = -1/2$

- $\tan(\sin^{-1}(1/5)) = \tan(\theta) = \dfrac{opp}{adj} = \dfrac{1}{\sqrt{24}}$

$adj = \sqrt{24} = 2\sqrt{6}$

$\theta = \sin^{-1}(1/5)$

$\sin(\theta) = \dfrac{1}{5} = \dfrac{opp}{hyp}$

$adj^2 + 1^2 = 5^2$

$adj^2 = 24$

- $\tan(\sin^{-1}x) = \dfrac{opp}{adj} = \dfrac{x}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}$

$\theta = \sin^{-1}x$

$\sin\theta = \dfrac{x}{1} = \dfrac{opp}{hyp}$

- $\cos(\sin^{-1}x) = \dfrac{adj}{hyp} = \dfrac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

- $\sec(\sin^{-1}x) = \dfrac{1}{\cos(\sin^{-1}x)} = \dfrac{1}{\sqrt{1-x^2}}$

- $\sin(\sin^{-1}x) = x$

$$\boxed{\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}}}$$

← $-1 < x < 1$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$$

for $-1 < x < 1$

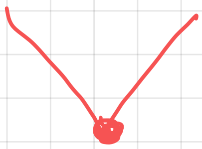
Problems

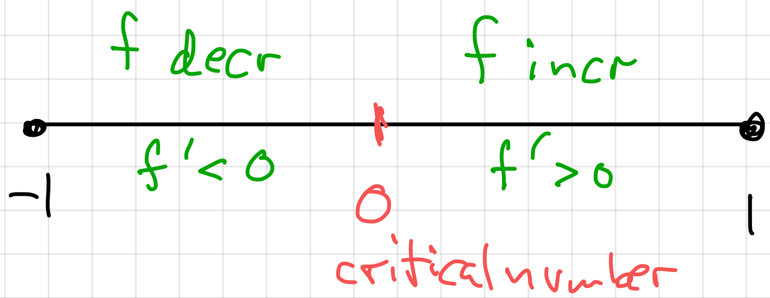
$f(0) = \arcsin(0) = 0$ is the minimum value

① Does $f(x) = \arcsin(x^2)$ have a minimum value ?? Yes

$$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}} \frac{d}{dx}\left[x^2\right] = \frac{2x}{\sqrt{1-x^4}}$$

only $= 0$ when $x = 0$





← domain(f) $= [-1, 1]$

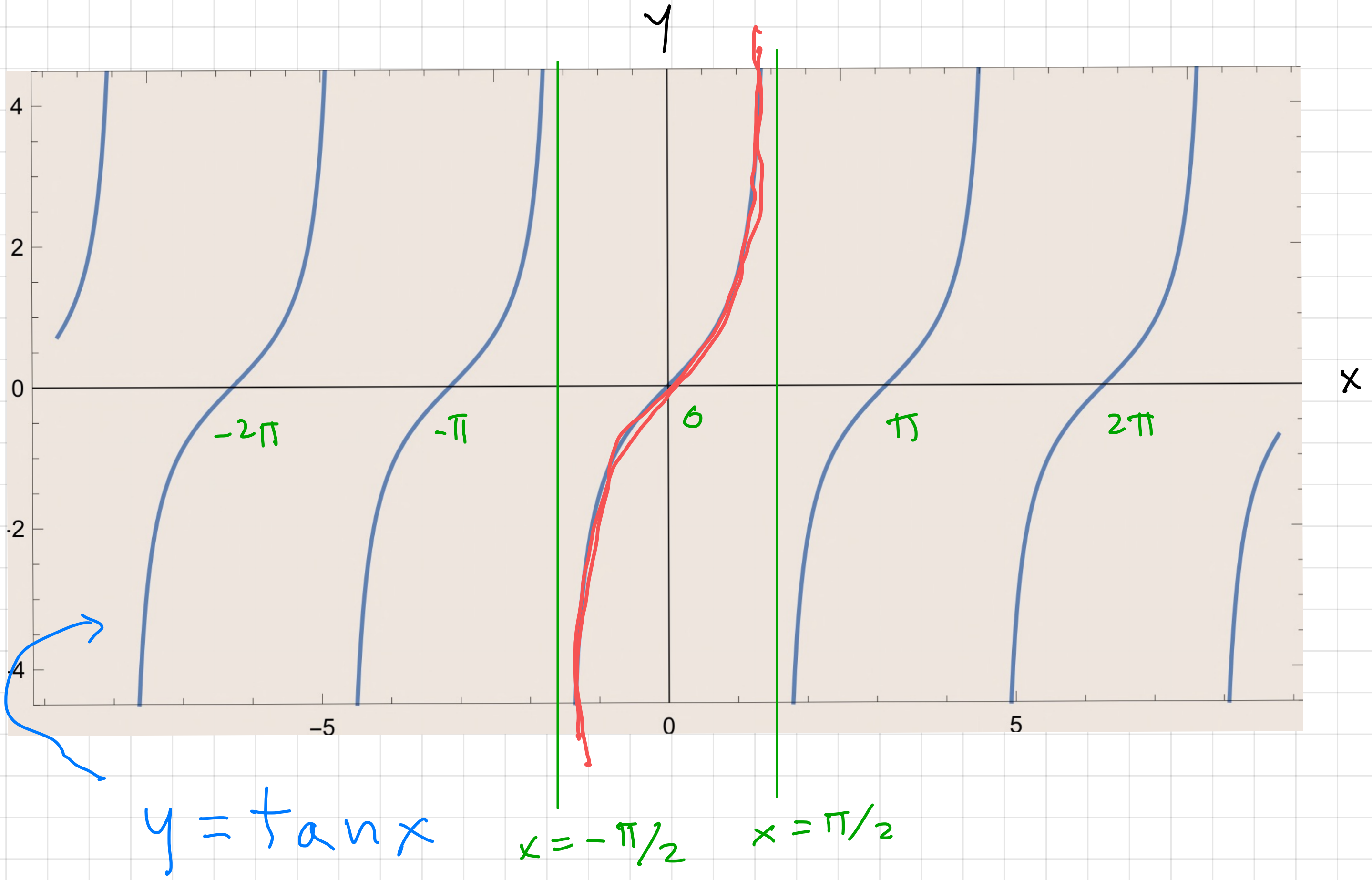
② $$\frac{1}{3}\int \frac{3x^2}{\sqrt{1-x^6}}\,dx$$

← try $\begin{cases} u = x^3 \\ du = 3x^2\,dx \end{cases}$

$x^6 = u^2$

$$= \frac{1}{3}\int \frac{du}{\sqrt{1-u^2}} =$$

$$= \frac{1}{3}\arcsin(u) + C = \frac{1}{3}\arcsin(x^3) + C$$



The function $f(x) = \tan(x)$, $-\pi/2 < x < \pi/2$ has an inverse function:

$$\tan^{-1}(x) = \arctan(x) = f^{-1}(x).$$

and domain(arctan) = range(f) = $(-\infty, \infty)$. So

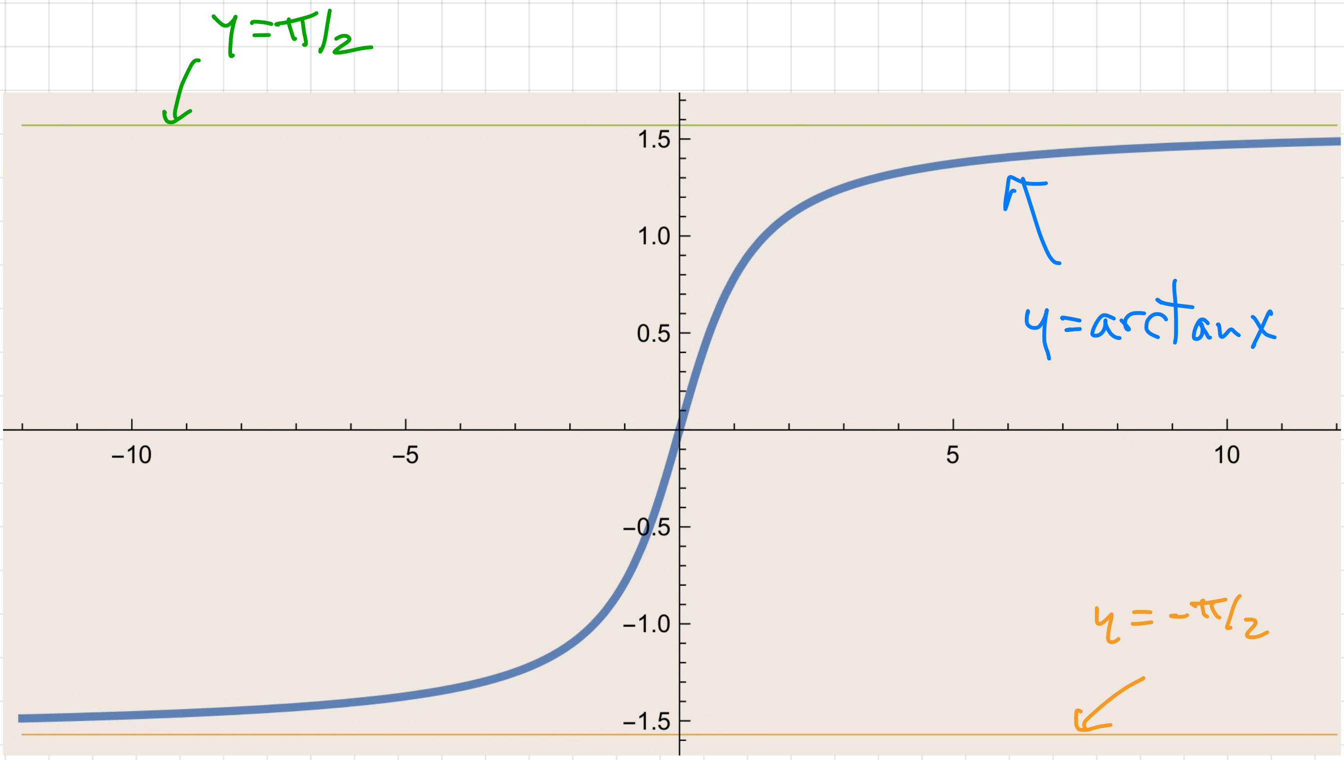
$\arctan(x) =$ the angle θ between $-\pi/2$ and $\pi/2$ for which $\tan\theta = x$

$$\boxed{\begin{array}{ll} \tan(\arctan x) = x & \text{for any real number } x \\ \arctan(\tan x) = x & \text{for } -\pi/2 < x < \pi/2 \end{array}}$$

examples

$\arctan(-1) = -\pi/4$

$\arctan(1/\sqrt{3}) = \pi/6$



$$\frac{d}{dx}\left[\tan(\arctan x)\right] = \sec^2(\arctan x)\frac{d}{dx}\left[\arctan x\right]$$

$$\parallel$$

$$\frac{d}{dx}[x] = 1$$

$$\Longrightarrow$$

$$\frac{d}{dx}\left[\arctan x\right] = \cos^2(\arctan x)$$

$$= \frac{1}{1+x^2}$$