True or False?

All are true!

① If $F(x)$ is an antiderivative of $f(x)$ then
$$\int_a^b f(x)\,dx = F(b) - F(a).$$
FTC

② $F(t) = \dfrac{t+1}{t^2+1}$ is an antiderivative of
$$f(t) = \frac{d}{dt}\left[\frac{t+1}{t^2+1}\right].$$
$F'(t) = f(t)$

③ $$\int_2^4 \frac{d}{dt}\left[\frac{t+1}{t^2+1}\right] dt = \underbrace{\frac{4+1}{4^2+1}}_{= F(4)} - \underbrace{\frac{2+1}{2^2+1}}_{= F(2)}$$
just use ① and ②

④ $$\int \frac{1}{\sqrt{1+x}\,\sqrt{1+\sqrt{1+x}}}\,dx = 4\sqrt{1+\sqrt{1+x}} + C$$

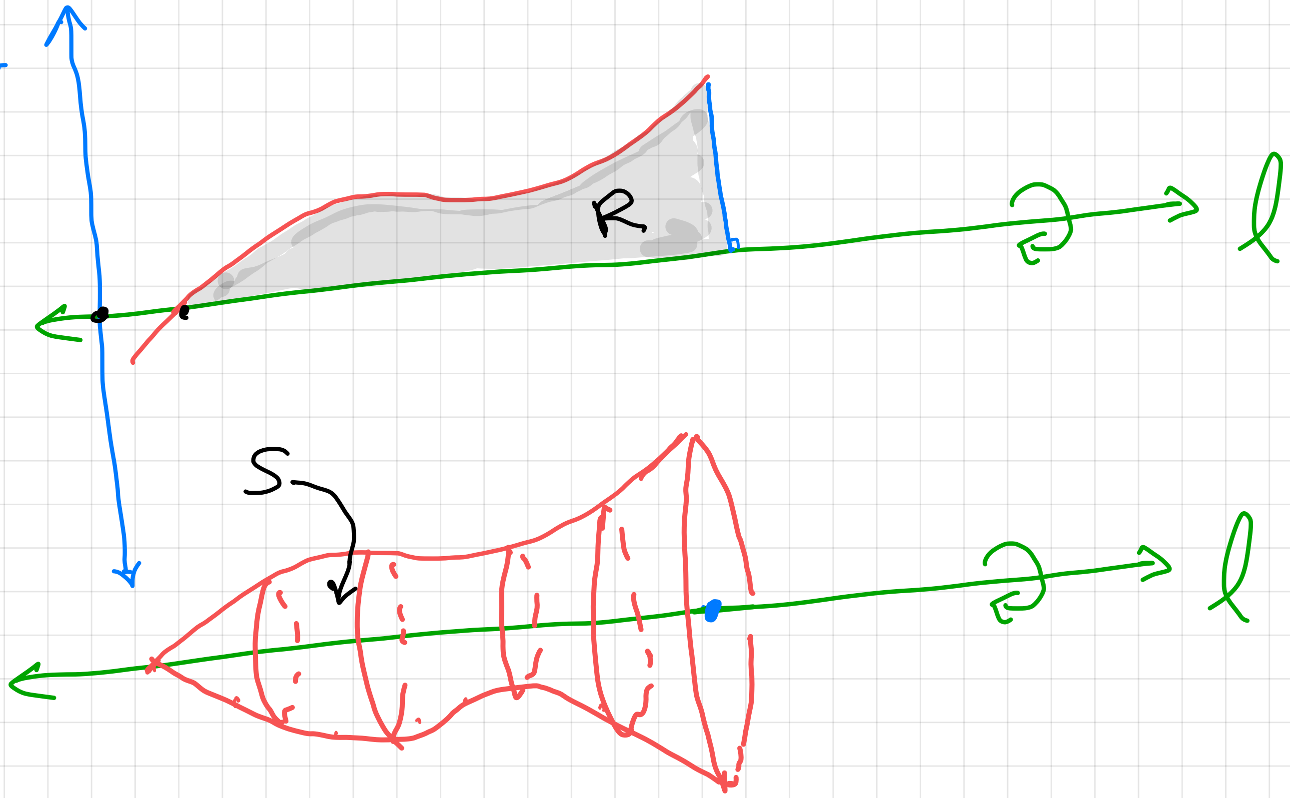
$$\frac{d}{dx}\left[4\left(1+(1+x)^{1/2}\right)^{1/2}\right] = 4\,\tfrac{1}{2}\left(1+(1+x)^{1/2}\right)^{-1/2}\frac{d}{dx}\left[1+(1+x)^{1/2}\right]$$
$$= \cancel{2}\,\frac{1}{\sqrt{1+\sqrt{1+x}}}\;\frac{1}{\cancel{2}}(1+x)^{-1/2}$$
$$= \frac{1}{\sqrt{1+\sqrt{1+x}}}\;\frac{1}{\sqrt{1+x}}$$

This checks that ④ is true.

(note: used the chain rule twice.)

VOLUME (sections 5.2 and 5.3)

A <u>solid of revolution</u> is constructed from a planar region R and a line ℓ in the same plane by rotating R around ℓ. The resulting solid S has ℓ as a "rotational axis of symmetry".



We will discuss two methods for finding the volume of S.

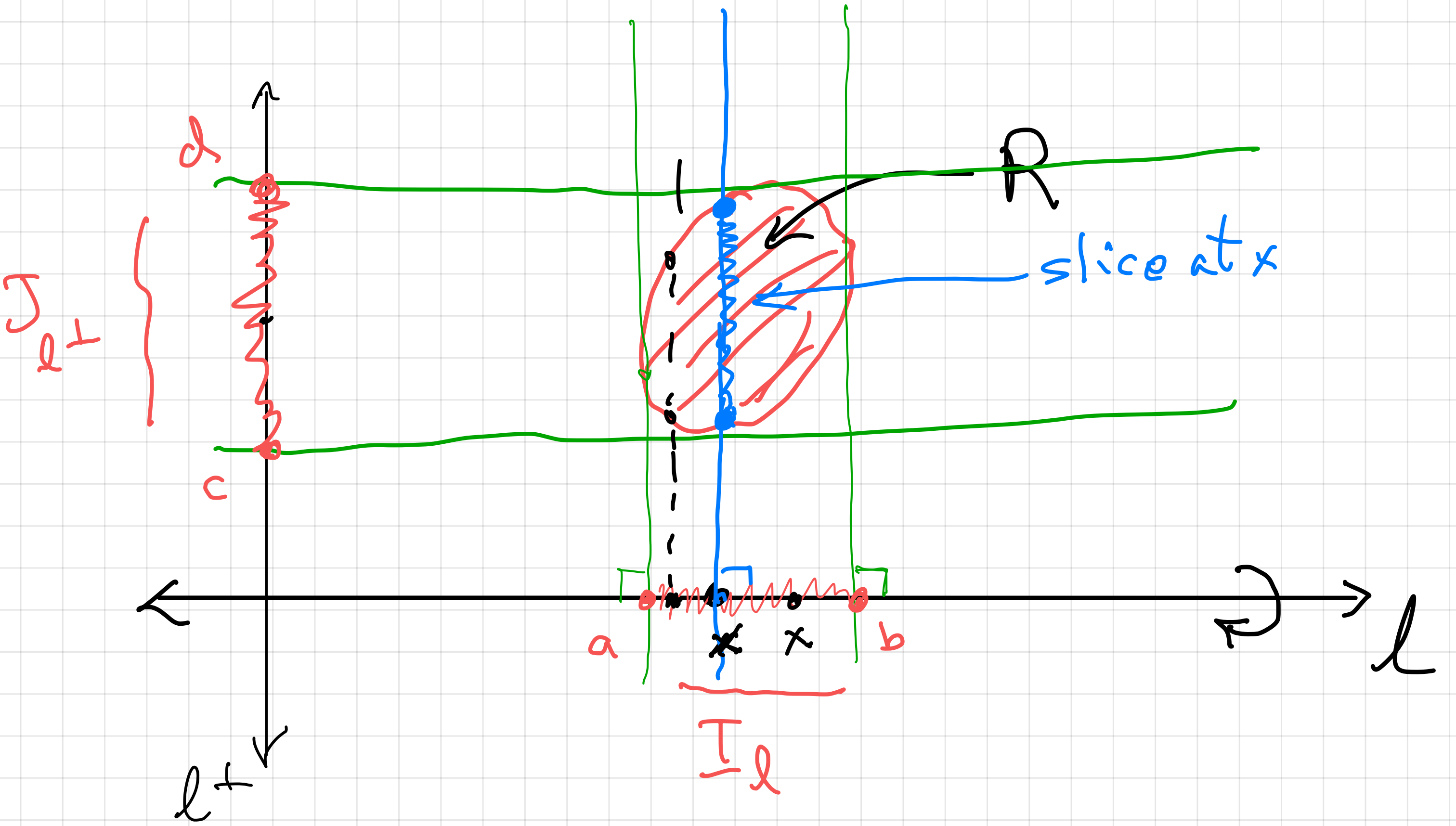
① Disk or Washer method:

use ℓ as reference line.

② Cylindrical shell method:

use a line $\ell^{\perp}$ perpendicular to ℓ as reference line.

$\perp$ = "perpendicular"

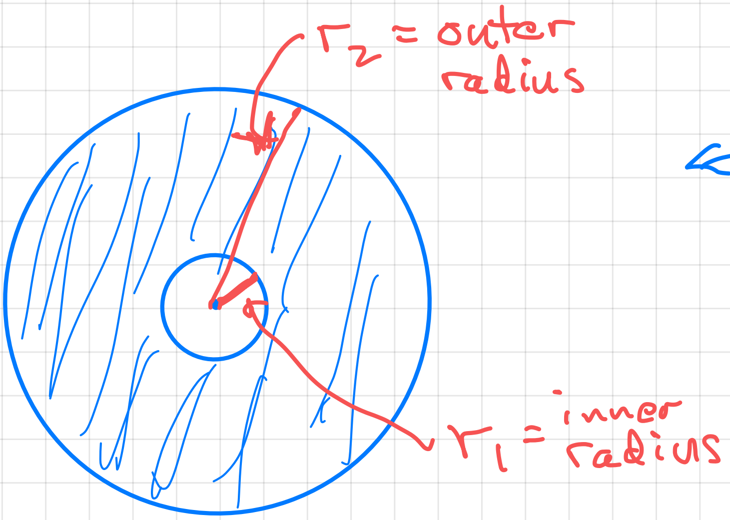


Assume that ℓ and $\ell^\perp$ are number lines.

I_ℓ = projection of R to ℓ.

$J_{\ell^\perp}$ = projection of R to $\ell^\perp$.

Rotate the slice at x around ℓ.



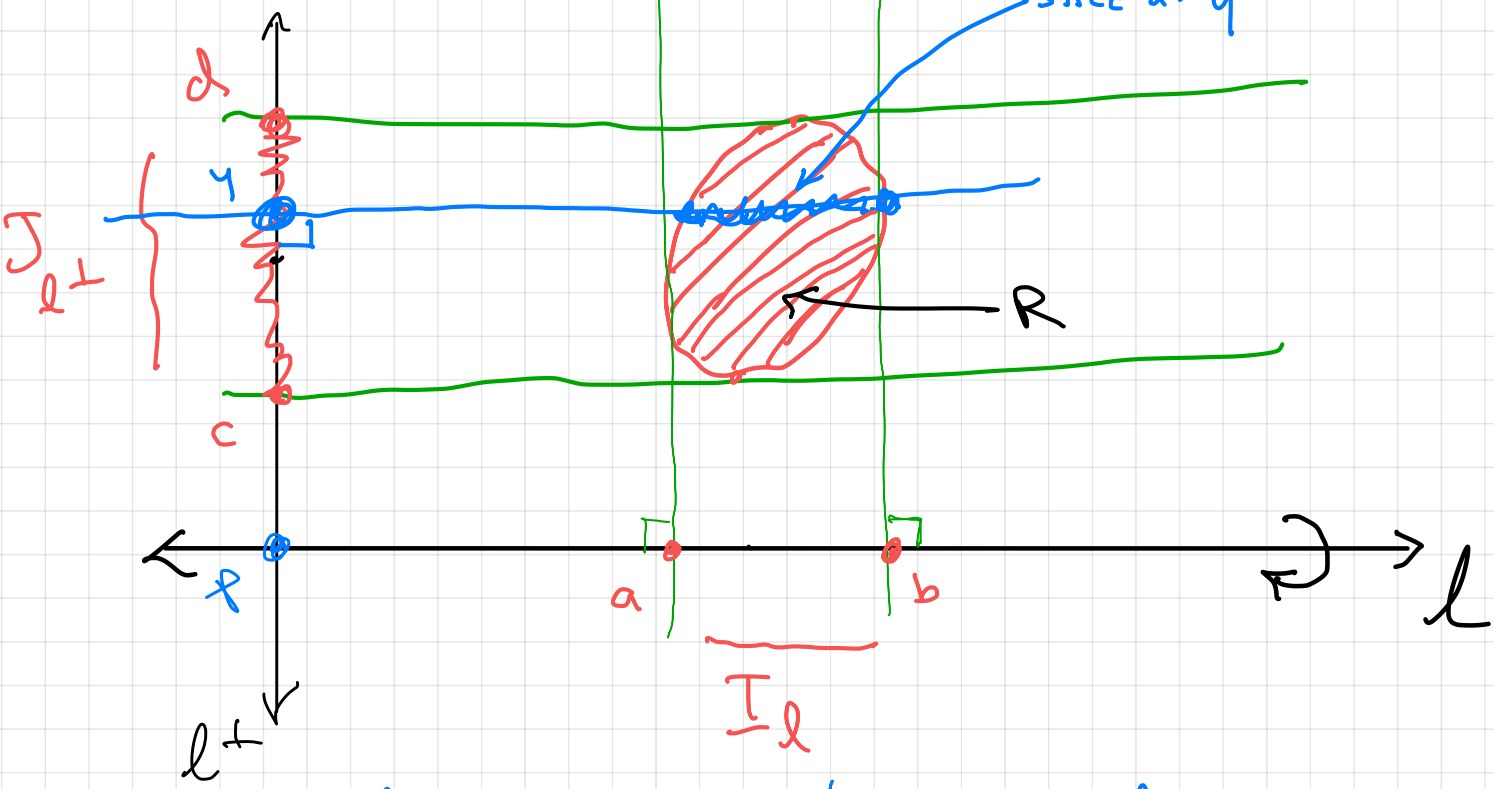
washer area $= \pi r_2^2 - \pi r_1^2$

← washer method

Theorem Volume of solid of revolution

$$= \int_a^b \text{area(washer at } x) \, dx$$

for y between c and d.



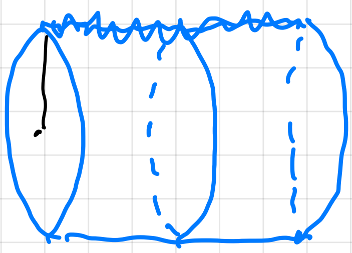
Shell method: Let y be between c and d.

The line thru y parallel to ℓ intersects the region in a slice. Rotate this slice around ℓ to get a cylinder.

Theorem Volume of solid $= \int_c^d \text{area}(\text{cylinder at } y)\, dy$

$w =$ length of slice at y

radius $r =$ distance $|\overline{yp}|$



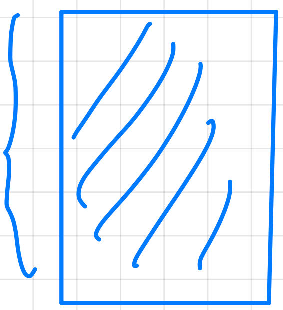
area cylinder $=$

$2\pi r w$

cut open

$= 2\pi\, |\overline{yp}| \cdot (\text{length of slice at } y)$

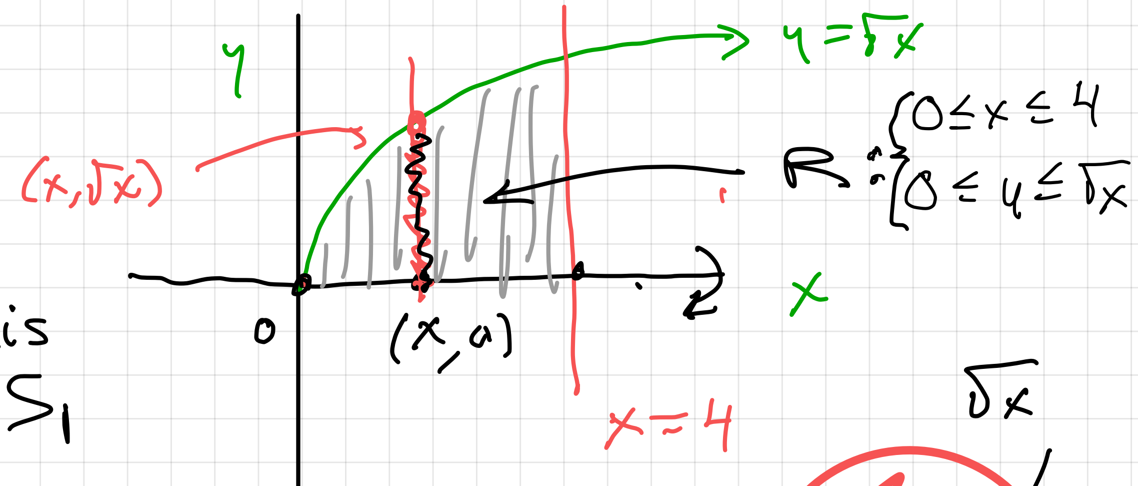
$w =$ width



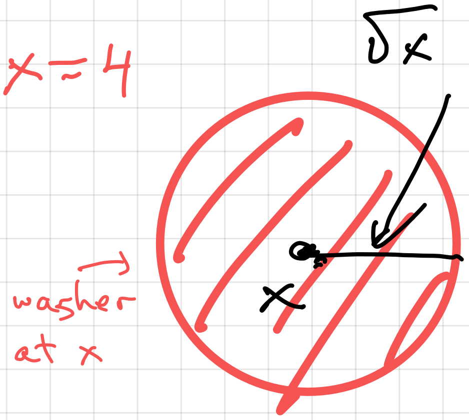
$2\pi r$ = circumference of circle

example 1

rotate R around x-axis to get solid S_1



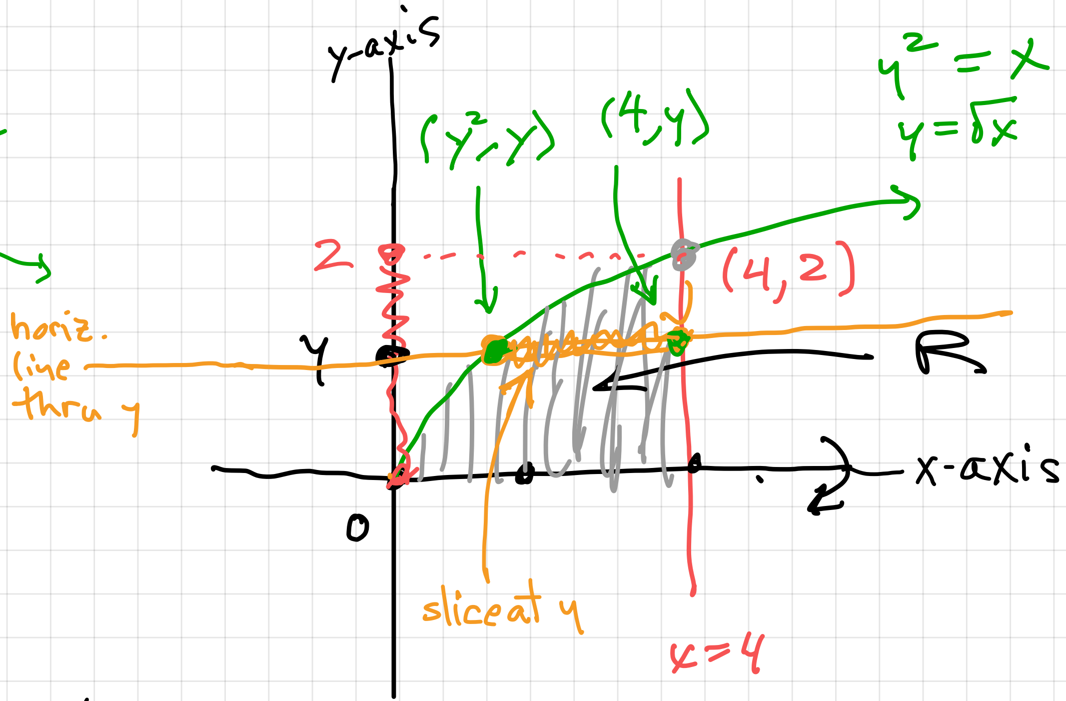
$$\text{Volume}(S_1) = \int_0^4 \pi (\sqrt{x})^2\, dx$$

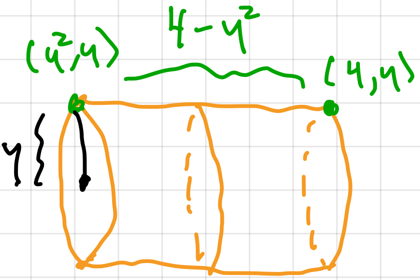


$$= \int_0^4 \pi x\, dx = 8\pi$$

washer method

shell method





cylinder (shell) has width $4-y^2$ and radius y $\Rightarrow$

$$\text{Volume}(S_1) = \int_0^2 2\pi y(4-y^2)\, dy$$

area of slice at y

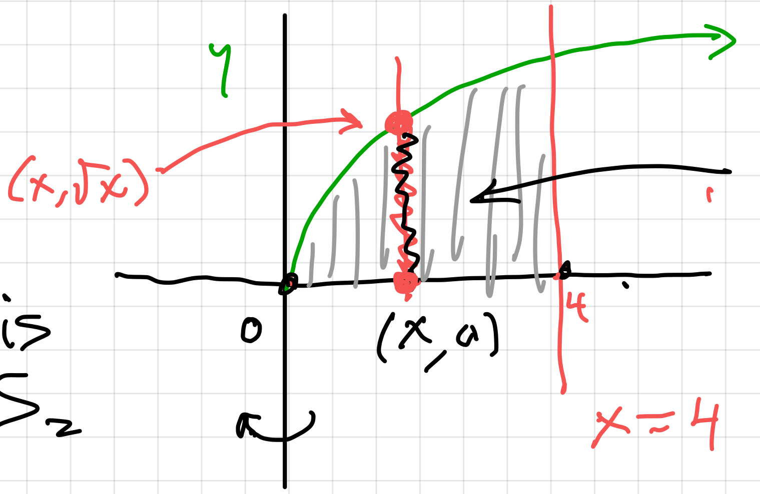
$$= \int_0^2 8\pi y - 2\pi y^3\, dy$$

$$= 8\pi \frac{y^2}{2} - 2\pi \frac{y^4}{4} \Big|_{y=0}^{2} = 8\pi \cdot 2 - 2\pi \frac{16}{4}$$

$$= 16\pi - 8\pi = 8\pi$$

example 2

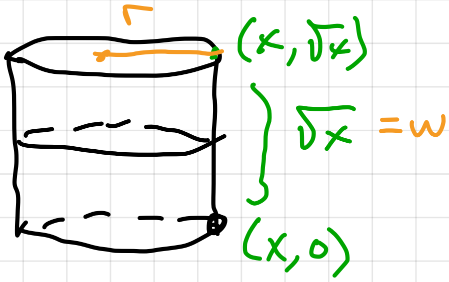
rotate R around y-axis to get solid S_2



$R:\begin{cases}0\le x\le 4\\ 0\le y\le \sqrt{x}\end{cases}$

shell method: reference line = x-axis, $0\le x\le 4$

cylinder at x =

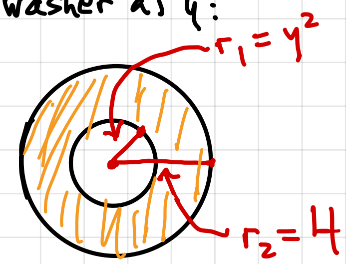


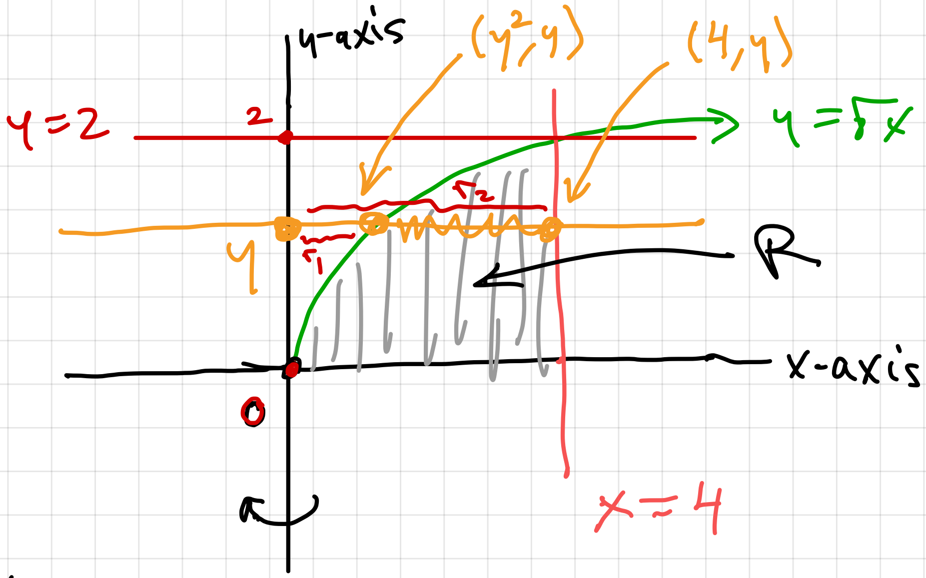
r = radius = x

$$\text{Volume}(S_2)=\int_0^4 \underbrace{2\pi x\sqrt{x}}_{2\pi r w}\,dx=2\pi\int_0^4 x^{3/2}\,dx=\frac{128}{5}\pi$$

washer method: reference line = y-axis : $0\le y\le 2$

washer at y:





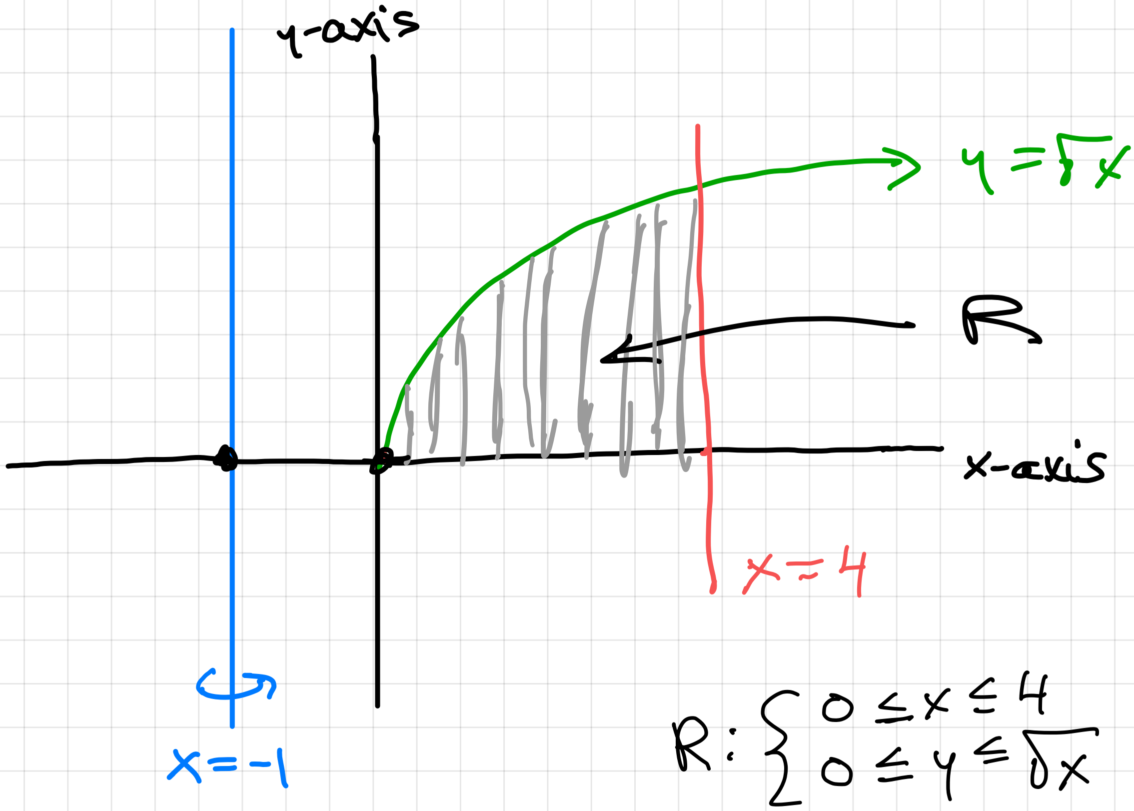
$\text{Volume}(S_2)=$

$$\int_0^2 \pi(4^2)-\pi(y^2)^2\,dy$$

$$=\pi\int_0^2 16-y^4\,dy=\pi\left(16y-\frac{1}{5}y^5\right)\Big|_0^2=\frac{128}{5}\pi$$

example 3

rotate R around the line $x=-1$ to get solid S_3



$$R: \begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$$

shell method:

$$\text{Volume}(S_3) = \int_0^4 2\pi(x+1)\sqrt{x}\,dx = \int_0^4 2\pi x^{3/2} + 2\pi x^{1/2}\,dx$$

$$= \frac{544}{15}\pi$$

washer method:

$$\text{Volume}(S_3) = \int_0^2 \pi 5^2 - \pi(y^2+1)^2\,dy$$

$$= \int_0^2 24\pi - 2\pi y^2 - \pi y^4\,dy = \frac{544}{15}\pi$$