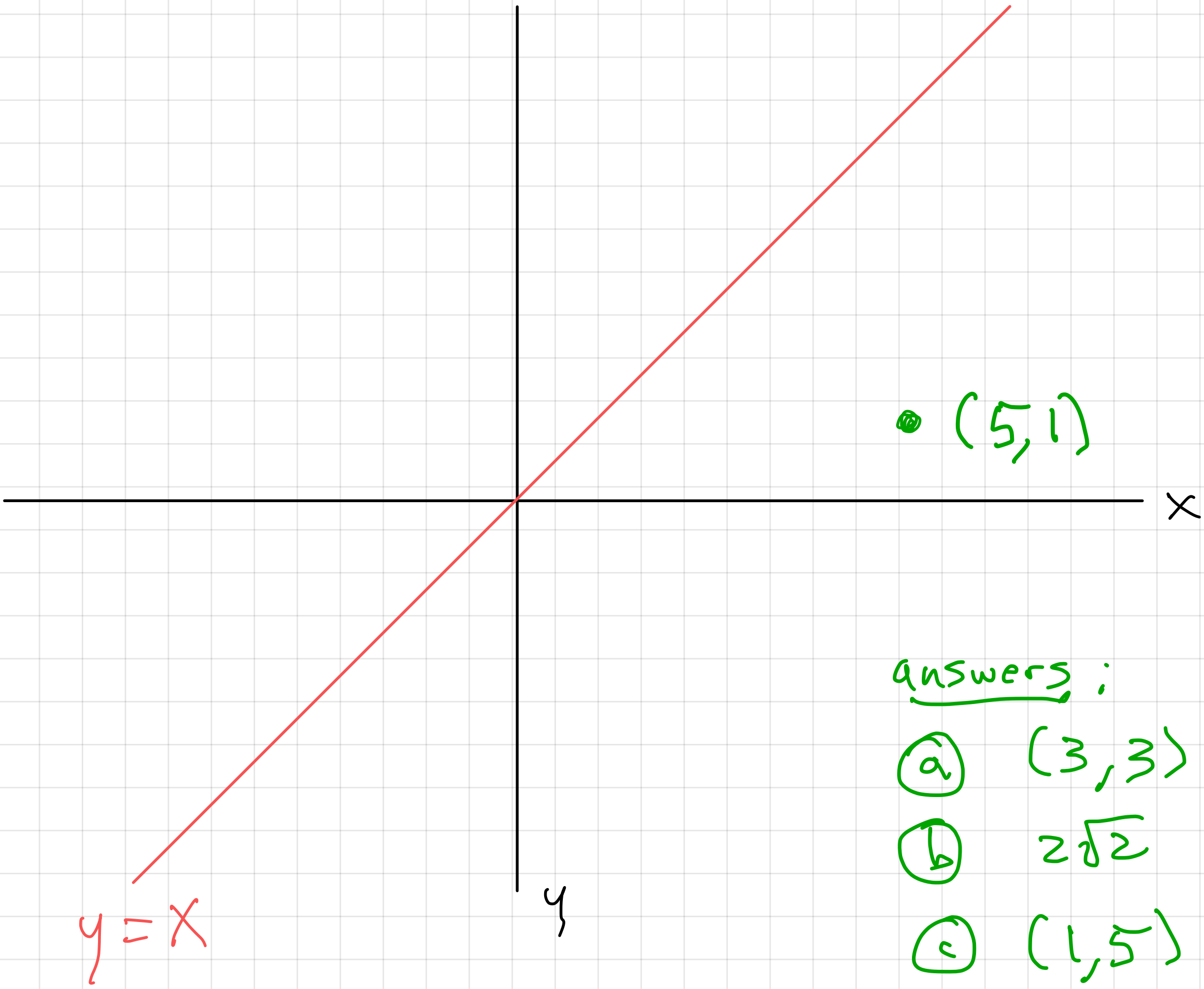
Questions:

① Which point on the line $y=x$ is closest to $(5,1)$?

② What is the shortest distance from $(5,1)$ to $y=x$?

③ What are the coordinates of the mirror image of $(5,1)$ across $y=x$?



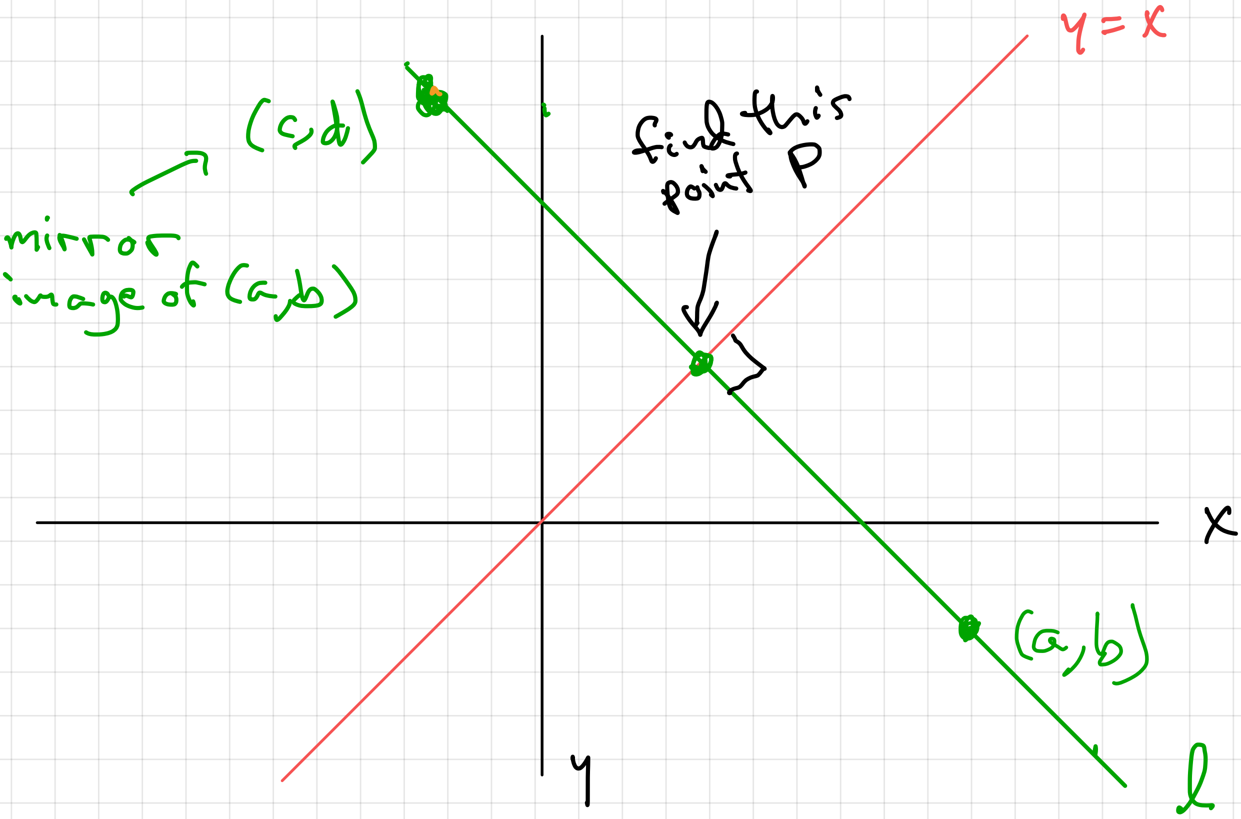
answers:

ⓐ $(3,3)$

ⓑ $2\sqrt{2}$

ⓒ $(1,5)$

To examine this we'll consider the more general problem where $(5,1)$ is replaced by an arbitrary point (a,b):



ℓ = line thru (a,b) perpendicular to $y=x$.

slope of ℓ = $\frac{1}{-1} = -1$

equation for ℓ: $y - b = (-1)(x-a)$

$$y = -x + (a+b)$$

POI of ℓ and $y=x$ is $P=(x,y)$ where

$x = y = -x + (a+b) \Rightarrow 2x = a+b$

So $\boxed{(x,y) = \left(\frac{a+b}{2}, \frac{a+b}{2}\right) = P}$

If (c,d) is the mirror point of (a,b) then:

$$P = \text{midpoint of } (a,b) \text{ and } (c,d) = \left(\frac{a+c}{2}, \frac{b+d}{2}\right)$$

so

$$\frac{a+b}{2} = \frac{a+c}{2} \Rightarrow c = b \text{ and } \frac{a+b}{2} = \frac{b+d}{2} \Rightarrow d = a$$

which shows that

$$\boxed{(c,d) = (b,a)}$$

About Exam 1:

The goal is to draw good schematic graphs to use to aid in solving problems.

Work on making your graphs more robust and accurate.

A little hard to follow your logic here. Try to work on organizing your explanations more clearly.

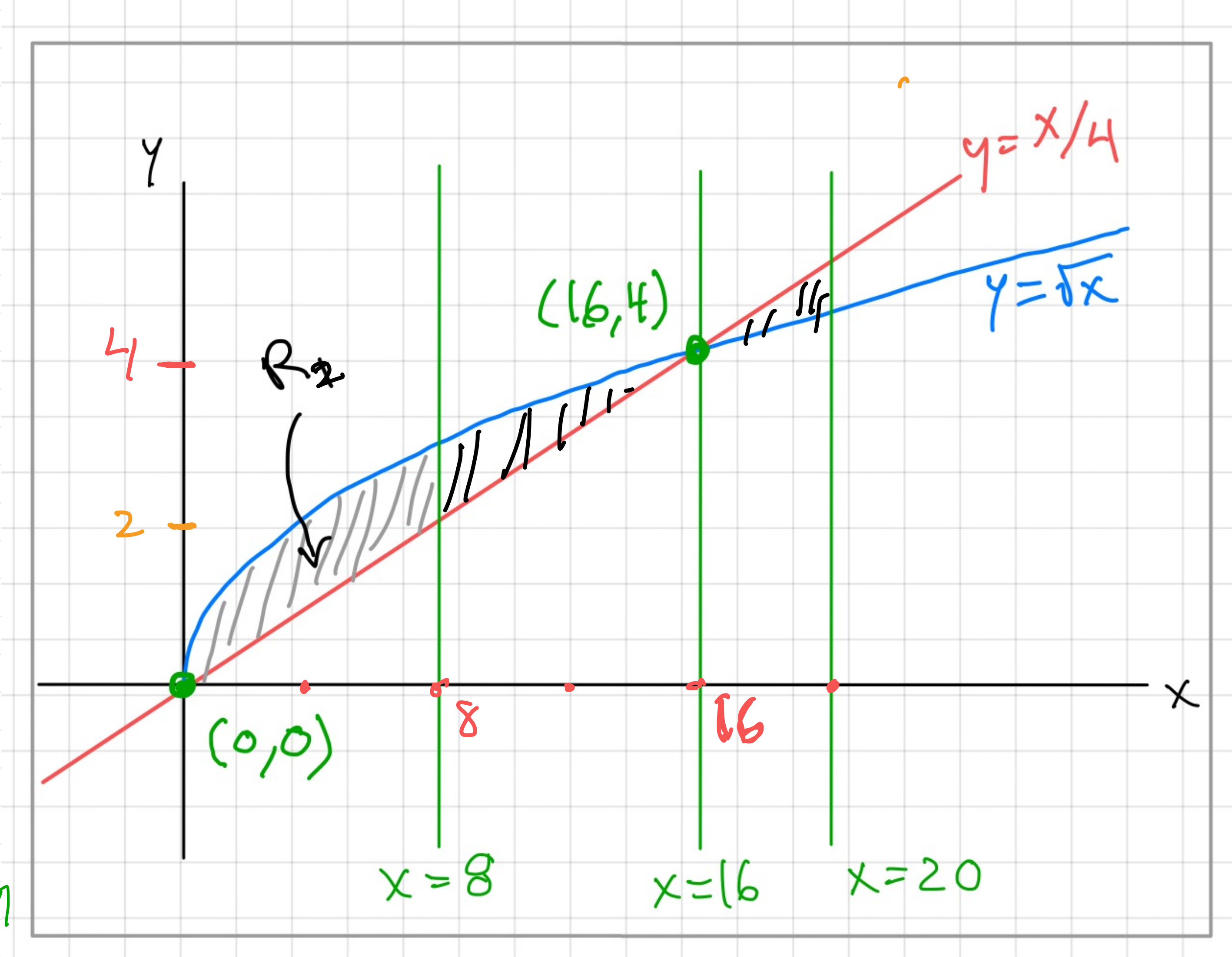
Work on making your logic more clear. This will be very important as problems get more intricate.

equal? Then you must write "=".

Does "$\longrightarrow$" mean "equals"? Write "=" then!

0 and 1 are limits for x, not for u.

Importance of well-drawn graphs.



Draw robust graphs with every element clearly labeled.

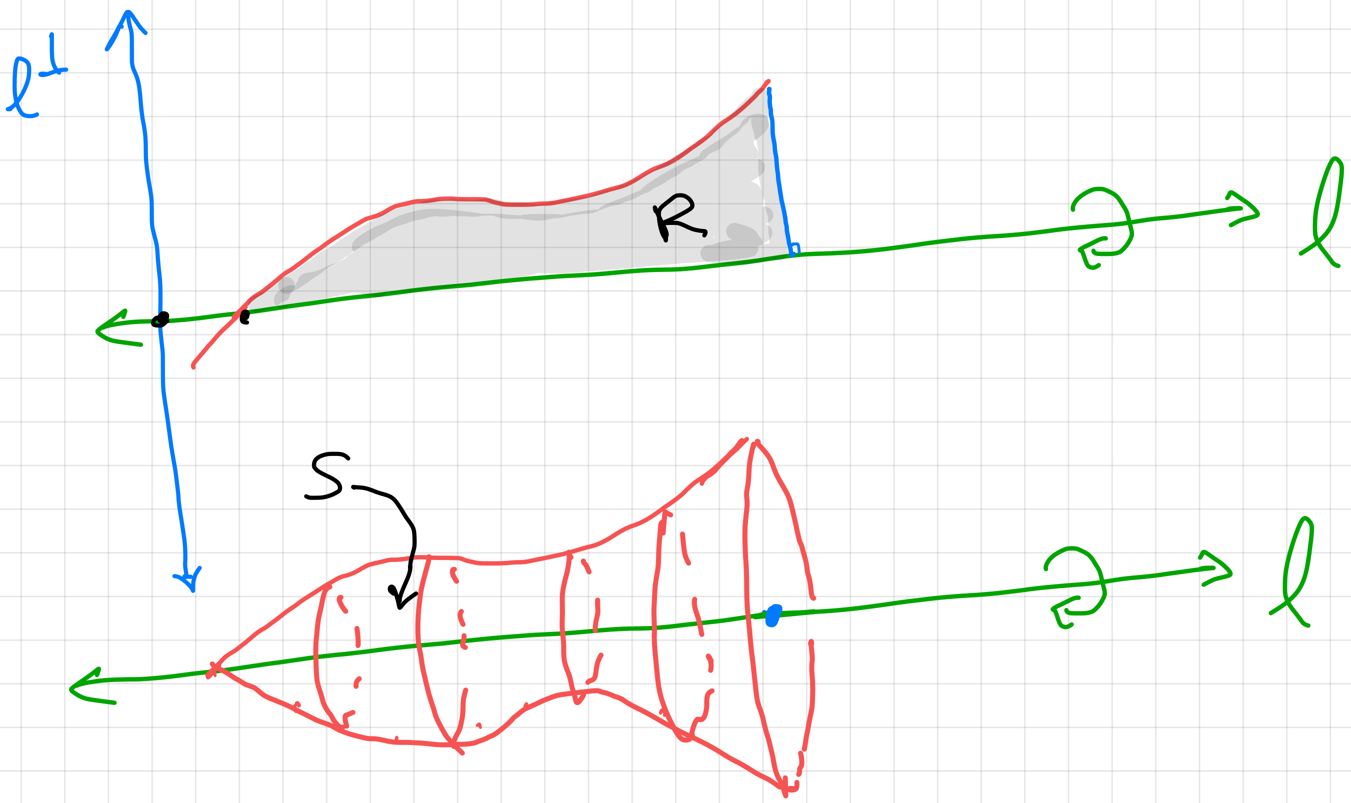
$y = x/4$ is a line thru (0,0) with slope $1/4$.

$y = \sqrt{x} \Rightarrow y^2 = x$ (half a parabola)

#1 (d) Let $\mathcal{R}_2$ be the region between the curves $y = f(x)$ and $y = g(x)$, and between the vertical lines $x = 0$ and $x = 20$. Use integration to calculate the area of $\mathcal{R}_2$.

VOLUME (sections 5.2 and 5.3)

A <u>solid of revolution</u> is constructed from a planar region R and a line ℓ in the same plane by rotating R around ℓ. The resulting solid S has ℓ as a "rotational axis of symmetry".



We will discuss two methods for finding the volume of S.

① **Disk or Washer method:**

use ℓ as reference line.

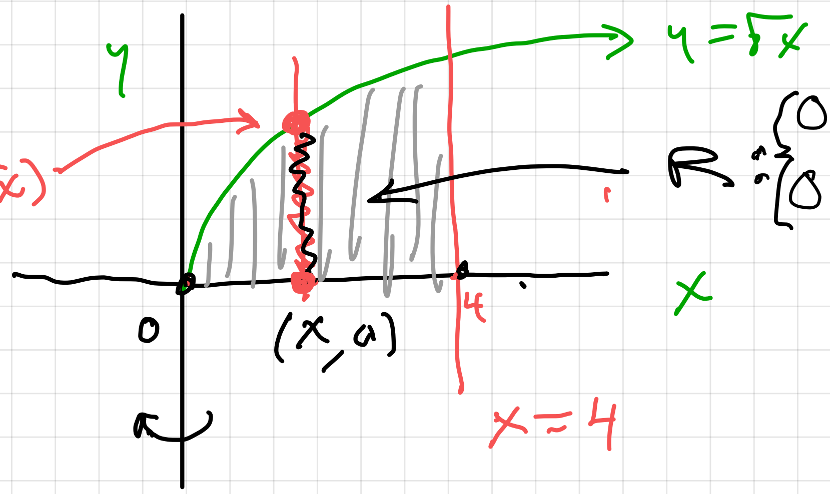
② **Cylindrical shell method:**

use a line $\ell^\perp$ perpendicular to ℓ as reference line.

$\perp$ = "perpendicular"

example 2

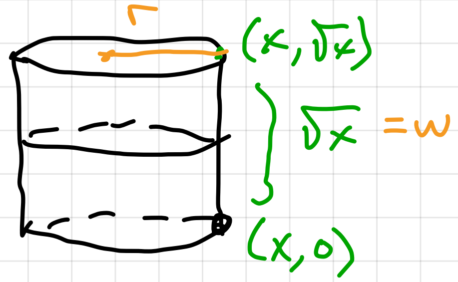
rotate R
around y-axis
to get solid S_2



$R: \begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$

shell method : reference line = x-axis, $0 \le x \le 4$

cylinder at x =



r = radius = x

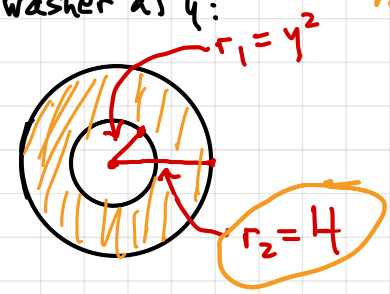
area cylinder $= 2\pi x \sqrt{x}$

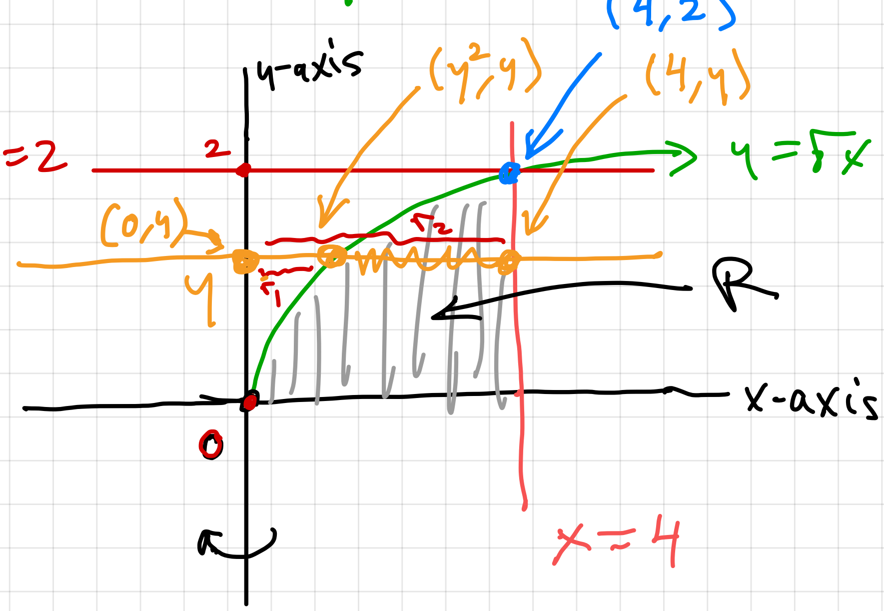
$$\text{Volume}(S_2) = \int_0^4 \underset{\underset{2\pi r w}{\parallel}}{2\pi x \sqrt{x}}\, dx = 2\pi \int_0^4 x^{3/2}\, dx = \frac{128}{5}\pi$$

washer method : reference line = y-axis : $0 \le y \le 2$

area washer = $\pi 4^2 - \pi (y^2)^2$

washer at y:





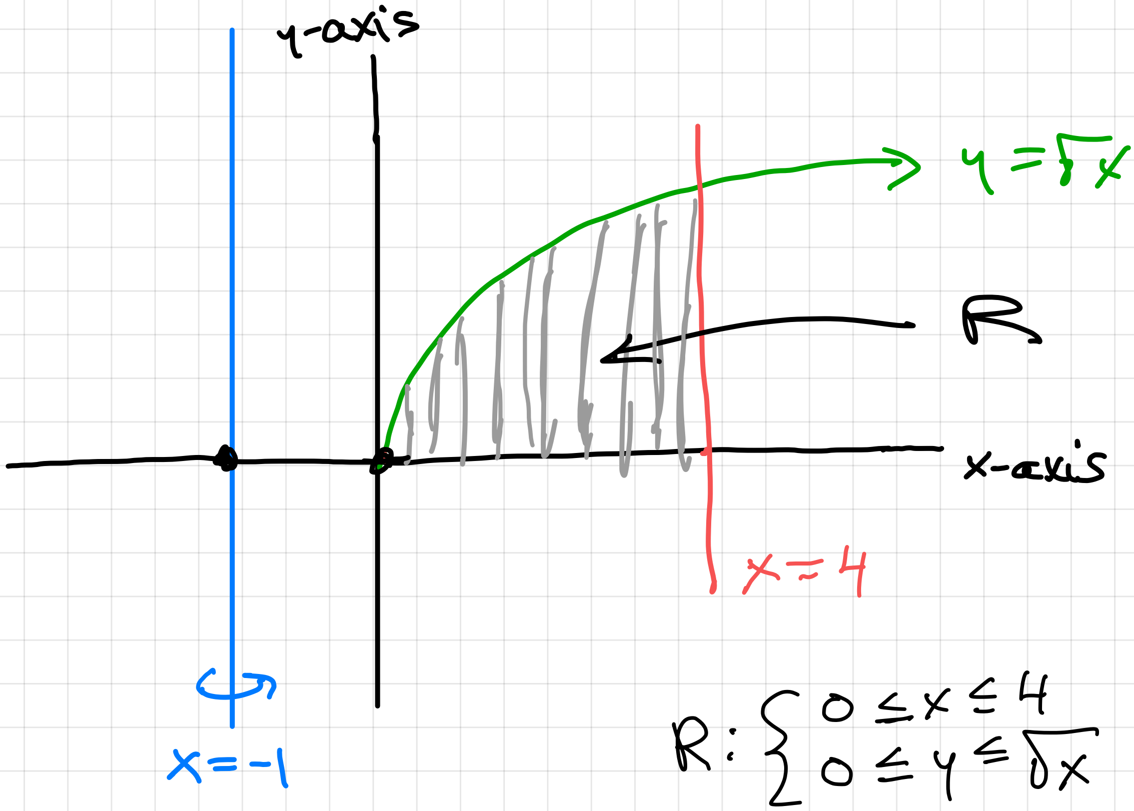
Volume(S_2) =

$$\int_0^2 \pi(4^2) - \pi(y^2)^2\, dy$$

$$= \pi \int_0^2 16 - y^4\, dy = \pi\left(16y - \frac{1}{5}y^5\right)\Big|_0^2 = \frac{128}{5}\pi$$

example 3

rotate R around the line $x=-1$ to get solid S_3



$$R:\begin{cases} 0\le x\le 4 \\ 0\le y\le \sqrt{x} \end{cases}$$

shell method:

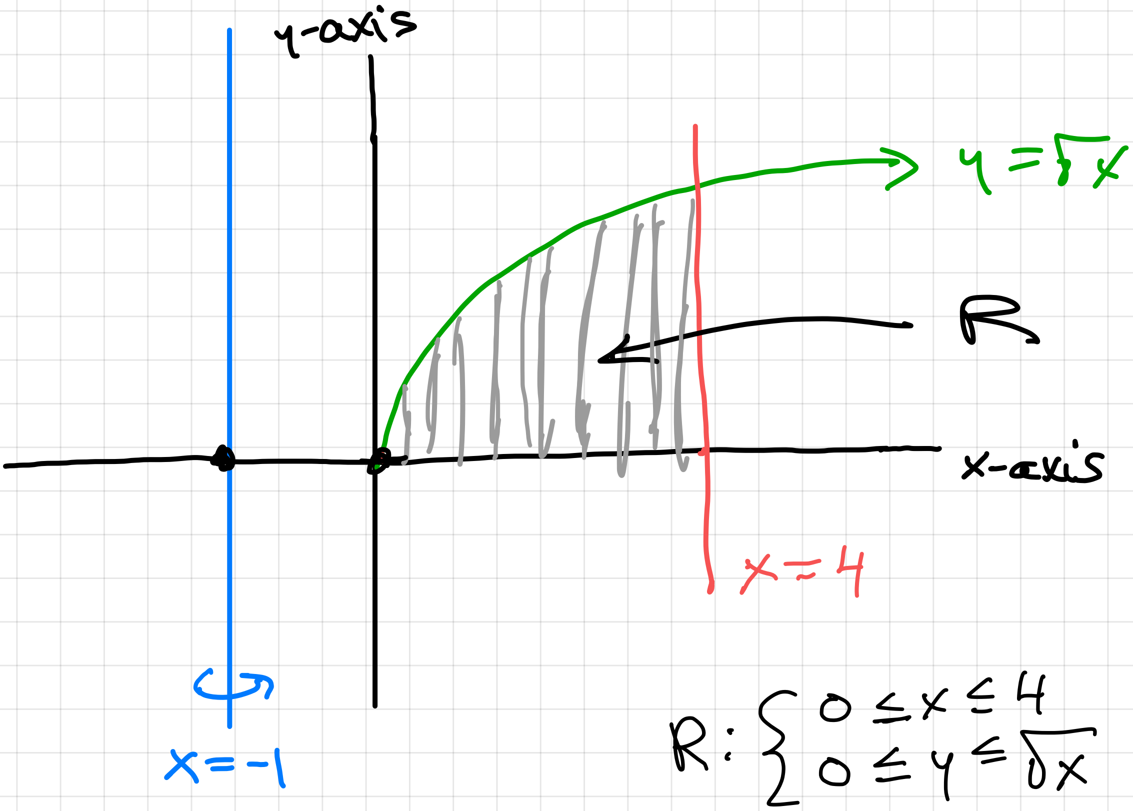
$$\text{Volume}(S_3)=\int_0^4 2\pi(x+1)\sqrt{x}\,dx=\int_0^4 2\pi x^{3/2}+2\pi x^{1/2}\,dx$$

$$=\frac{544}{15}\pi$$

washer method:

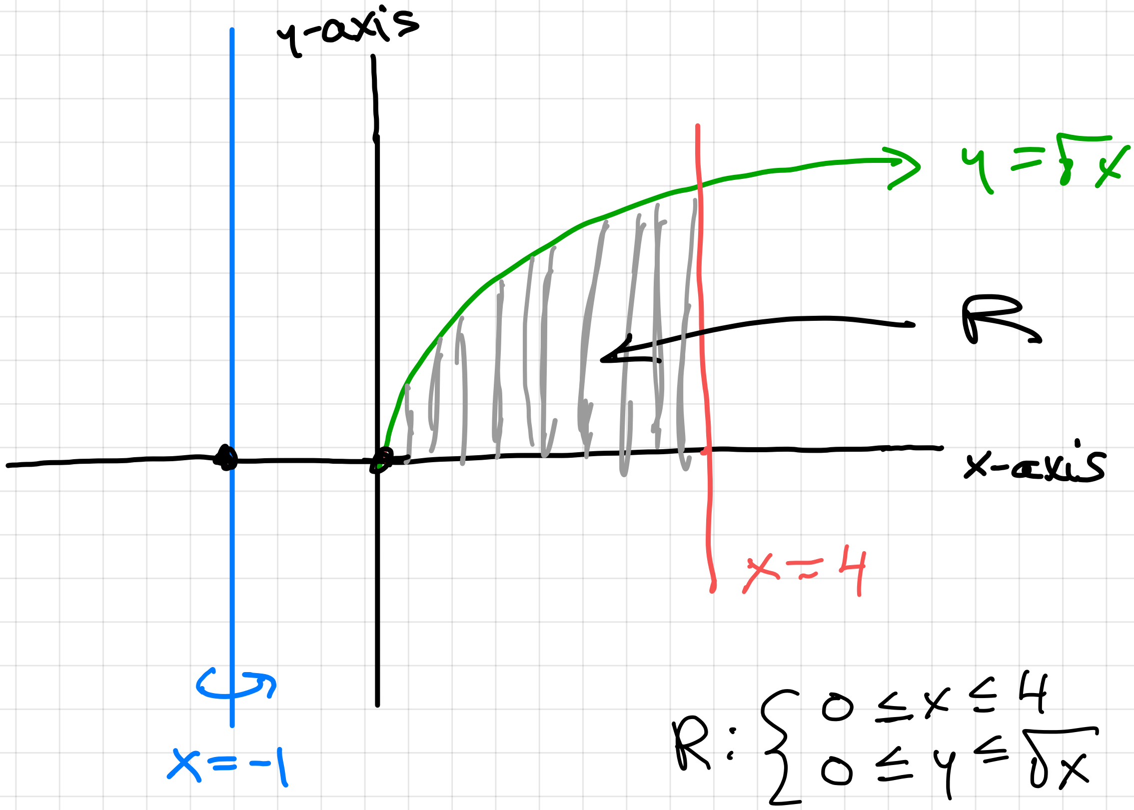
$$\text{Volume}(S_3)=\int_0^2 \pi 5^2-\pi(y^2+1)^2\,dy$$

$$=\int_0^2 24\pi-2\pi y^2-\pi y^4\,dy=\frac{544}{15}\pi$$



$$R: \begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$$

shell method:



$$R: \begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$$

Disk method:

Integrals of Some Basic Functions

- $\int x^p \, dx = \frac{1}{p+1} x^{p+1} + C$ if $p \neq -1$.

  why? b/c $\frac{1}{p+1}$ is undefined when $p = -1$

- $\int \sin x \, dx = -\cos x + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \sec^2(x) \, dx = \tan(x) + C$
- $\int \sec x \tan x \, dx = \sec x + C$

General Rule of Thumb!

Differentiation is easy.

Integration is hard.

(But both have lots of important applications.)

Chapter 6 goal

Add more basic functions to this list:

- logarithmic functions
- exponential functions
- inverse trig functions

<u>example:</u> $f(x) = \dfrac{x-2}{3x+1}$

Is there a number x so that $f(x)=10$? <u>Yes</u>

If so, how many x's are there? <u>Only one</u>

$$\frac{x-2}{3x+1} = 10 \quad \text{try to solve for } x$$

$$x-2 = 10(3x+1) = 30x+10$$

$$-12 = 29x \Rightarrow x = -12/29$$

Given a number y

Is there a number x so that $f(x)=y$?

If so, how many x's are there?

$$\frac{x-2}{3x+1} = y$$

$$x-2 = y(3x+1) = 3yx+y$$

$$-y-2 = 3yx-x = x(3y-1)$$

$$\Rightarrow \quad x = \frac{-y-2}{3y-1} = \frac{y+2}{1-3y}$$

If $y \neq 1/3$ then there is one and only one solution!

This shows that the functions :

$$f(x) = \frac{x-2}{3x+1}$$

and

$$g(y) = \frac{y+2}{1-3y}$$

are strongly linked together. We say they are a pair of "inverse functions".