Exam 1 – Take-Home Component **Name:**
Math 2423, 2/23/21

Instructions: Please print out this pdf document and write your answers in the space that is provided adding any extra sheets of paper that you may to complete your solutions. This cover sheet should be signed and submitted to validate your exam paper.

To get full credit on your work you must clearly indicate the thought processes that you have used to solve each problem. Please draw robust sketches with all of the prominent features clearly labeled.

Exam Procedures: In working on this Exam you are permitted to refer to Stewart's textbook or to the class lecture notes, and you are allowed to use a numerical calculator if desired. However you are not allowed to use any graphics capabilities on a calculator or any other device. You are not to discuss any of the problems on this exam with classmates or any other individuals until after the due date for the test has passed. No internet use (with the exception of accessing class notes) or use of any reference other than indicated above is allowed.

Please sign your name here to indicate that you have conformed to the procedures described above:

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Answers

Problem 1. Let $f(x) = \sqrt{x}$ and $g(x) = x/4$.

(a) In one picture, sketch the graphs of these two functions.

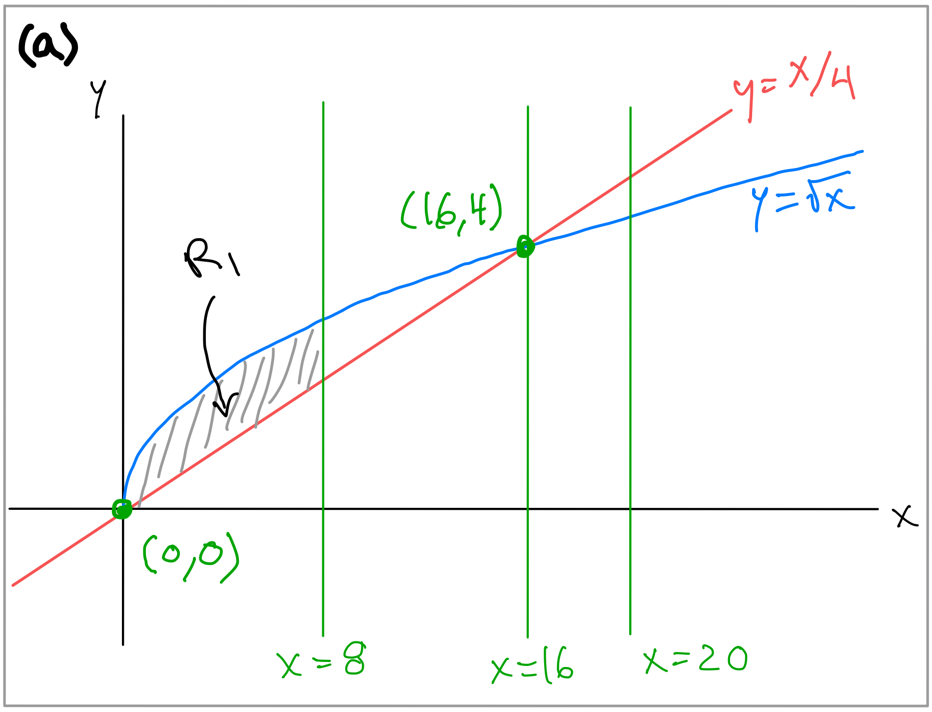
(b) Show the algebra that is necessary to determine any points of intersection between the two curves, and clearly label the coordinates of these points in your graph in part (a).

(c) Let $\mathcal{R}_1$ be the region between the curves $y = f(x)$ and $y = g(x)$, and the vertical lines $x = 0$ and $x = 8$. Use integration to calculate the area of $\mathcal{R}_1$.

(d) Let $\mathcal{R}_2$ be the region between the curves $y = f(x)$ and $y = g(x)$, and between the vertical lines $x = 0$ and $x = 20$. Use integration to calculate the area of $\mathcal{R}_2$.

(e) Find all possible positive numbers b for which the average values of $f(x)$ and $g(x)$ on the interval $[0, b]$ are equal.

Draw robust graphs with every element clearly labeled.



$y=\sqrt{x}$ is half of the parabola $x=y^2$ which has vertex at $(0,0)$ and opens to the right.

$y = x/4$ is the straight line of slope $1/4$ through $(0,0)$

(b) $x/4 = \sqrt{x} \Rightarrow x = 4\sqrt{x} \Rightarrow x^2 = 16x \Rightarrow x^2 - 16x = 0 \Rightarrow x(x-16) = 0$

So $x = 0$ or $x = 16$, and the points of intersection are $(0,0)$ and $(16,4)$

(c) $\text{Area}(R_1) = \int_0^8 x^{1/2} - \frac{1}{4}x\,dx = \frac{2}{3}x^{3/2} - \frac{1}{8}x^2 \Big|_0^8 = (32\sqrt{2} - 24)/3$

(d) $\text{Area}(R_2) = \int_0^{16} x^{1/2} - \frac{1}{4}x\,dx + \int_{16}^{20} \frac{1}{4}x - x^{1/2}\,dx = \underline{32} + \frac{2}{3}(91 - 40\sqrt{5})$

(see next page)

(e) On the interval $[0,b]$

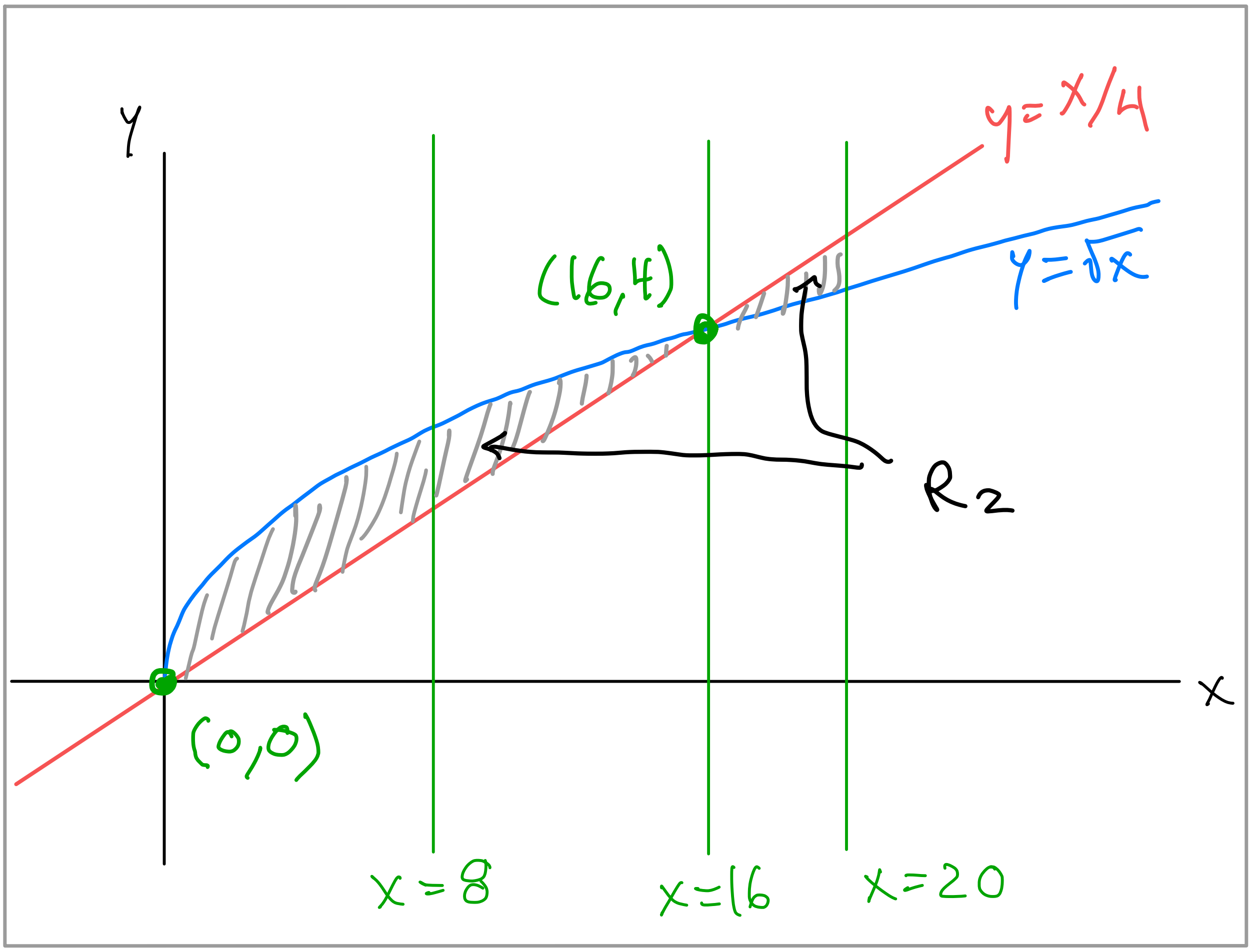
$$f_{ave} = \frac{1}{b}\int_0^b x^{1/2}\,dx = \frac{2}{3}\sqrt{b}$$, and

$$g_{ave} = \frac{1}{b}\int_0^b \frac{1}{4}x\,dx = \frac{b}{8}$$

So $f_{ave} = g_{ave}$ when $\frac{2}{3}\sqrt{b} = \frac{b}{8}$, and solving for b gives

$$b = 256/9$$ (since $b > 0$)

picture of R_2:



area of R_2 left of $x = 16$ $= \displaystyle\int_0^{16} \sqrt{x} - \frac{1}{4}x \, dx$

area of R_2 right of $x = 16$ $= \displaystyle\int_{16}^{20} \frac{1}{4}x - \sqrt{x} \, dx$

PROBLEM 2. Calculate each of the following integrals clearly indicating any substitution that you use:

(a) $\int 5\,dx$ $= 5x + C$

write substitution data clearly!

(b) $\int 5x^2 + 1\,dx$ $= \frac{5}{3}x^3 + x + C$

$$\boxed{\text{substitute } \begin{cases} u = 5x^2+1 \\ du = 10x\,dx \end{cases}}$$

(c) $\int x\cos(5x^2+1)\,dx$ $= \frac{1}{10}\int \cos(u)\,du =$

$= \frac{1}{10}\sin(5x^2+1) + C$

$$\boxed{\text{substitute } \begin{cases} v = \sin(u) \\ dv = \cos(u)\,du \end{cases}}$$

(d) $\int \frac{x\cos(5x^2+1)}{\sqrt{\sin(5x^2+1)}}\,dx$ $= \frac{1}{10}\int \frac{\cos(u)}{\sqrt{\sin(u)}}\,du =$

$= \frac{1}{10}\int \frac{1}{\sqrt{v}}\,dv = \frac{1}{5}v^{1/2} + C = \frac{1}{5}\sqrt{\sin(u)} + C = \frac{1}{5}\sqrt{\sin(5x^2+1)} + C$

(e) $\int \sec(\theta)\tan(\theta)\,d\theta$ $= \sec\theta + C$

$$\boxed{\begin{cases} u = x^3+1 \\ du = 3x^2\,dx \\ u(0) = 1 \\ u(1) = 2 \end{cases}}$$

(f) $\int_0^1 x^2(x^3+1)^6\,dx$ $= \frac{1}{3}\int_1^2 u^6\,du$

$= \frac{1}{21}u^7\Big|_{u=1}^{2} = \frac{127}{21}$

(g) $\int_2^4 \frac{d}{dt}\left[\frac{t+1}{t^2+1}\right]dt$ $= \frac{t+1}{t^2+1}\Big|_2^4 = \frac{5}{17} - \frac{3}{5} = -\frac{26}{85}$