From Stewart, page 148:

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

When you memorize this table, it is helpful to notice that the minus signs go with the derivatives of the "cofunctions," that is, cosine, cosecant, and cotangent.

2433/8-30

Scalar Multiplication

input: 1 vector, 1 scalar
output: 1 vector

If t is a scalar and $\vec{a} = \langle a_1, a_2, a_3 \rangle$ then $t\vec{a} = \langle ta_1, ta_2, ta_3 \rangle$.

example: Some scalar multiples of $\vec{a} = \langle 3, -1 \rangle$:

Geometric Interpretation: Two non-zero vectors $\vec{a}$ and $\vec{b}$ are parallel if and only if $\vec{b} = t\vec{a}$ for some scalar t.

Example $\langle 6, -8, 4 \rangle$ and $\langle -15, 20, -20 \rangle$ are not parallel because there is no solution t to the equations

$$\begin{cases} 6t = -15 \\ -8t = 20 \\ 4t = -20 \end{cases} \leftarrow \begin{cases} t = -5/2 \\ t = -5/2 \\ t = -5 \end{cases}$$

Math 2433-005
In-Class Quiz 8-30

Problem 1. In $\mathbb{R}^2$, consider the three points $P = (-1, 9)$, $Q = (1, 5)$ and $R = (7, -7)$ and the two vectors $\vec{a} = \overrightarrow{PQ}$ and $\vec{b} = \overrightarrow{QR}$.

(a) Sketch a graph of the xy-plane showing the three points and the two vectors.
(b) Express the vectors $\vec{a}$ and $\vec{b}$ in algebraic form. (ie– if $\vec{a} = \langle a_1, a_2 \rangle$ what are the values for a_1 and a_2. Similar for $\vec{b}$.)
(c) Is $\vec{b}$ a scalar multiple of $\vec{a}$? Use your results from part (b) to explain your answer.
(d) Are the three points P, Q and R colinear? Use part (c) to explain.

Problem 2. This problem refers to the same points P, Q and R, and vector $\vec{a}$ and $\vec{b}$ as in the previous problem.

(a) Determine the magnitudes of the vectors $\vec{a}$, $\vec{b}$ and $\overrightarrow{PR}$.
(b) Is the sum of two of those magnitudes equal to the third magnitude?

Some discussion:

$\overrightarrow{PQ} = \langle (1-(-1)), 5-9 \rangle = \langle 2, -4 \rangle \quad = \vec{a}$

$\overrightarrow{QR} = \langle 7-1, -7-(5) \rangle = \langle 6, -12 \rangle \quad = \vec{b}$

Does $\vec{b} = t\vec{a}$ for some scalar t? Yes take $t = 3$

b/c $3\vec{a} = 3\langle 2, -4 \rangle = \langle 6, -12 \rangle = \vec{b}$

(or write $\langle 6, -12 \rangle = t\langle 2, -4 \rangle = \langle 2t, -4t \rangle$ and try to solve for t: $6 = 2t$ and $-12 = -4t$. These equations do have a simultaneous solution $t = 3$.)

Conclude the vectors $\vec{a}$ and $\vec{b}$ parallel, and this means that the three points P, Q and R are collinear. At this point you should go back to your picture in (a) to make sure it conforms with this!!

Here are some basic properties of vector addition and scalar multiplication:

page 842:

Properties of Vectors If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in V_n and c and d are scalars, then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$
4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
7. $(cd)\mathbf{a} = c(d\mathbf{a})$
8. $1\mathbf{a} = \mathbf{a}$

Length ≡ magnitude ≡ norm

input: vector
output: scalar ≥ 0

If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ then

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

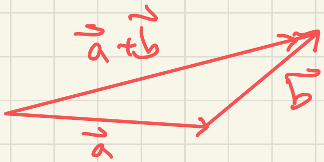
This is called the length (or magnitude, or norm) of $\vec{a}$.

Note The only vector $\vec{a}$ with $|\vec{a}| = 0$ is $\vec{a} = \vec{0} = \langle 0, 0, 0 \rangle$

Every other vector has positive length.

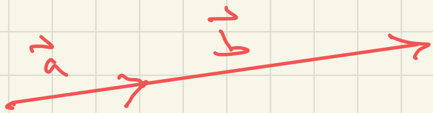
Geometric meaning of $|\vec{a}|$ should be obvious:

Example Its usually true that $|\vec{a} + \vec{b}| \neq |\vec{a}| + |\vec{b}|$



← In this picture, $|\vec{a} + \vec{b}| < |\vec{a}| + |\vec{b}|$

but here



← $|\vec{a}| + |\vec{b}| = |\vec{a} + \vec{b}|$