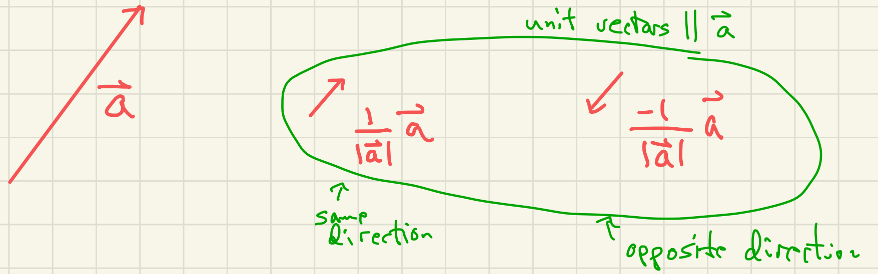
Unit Vectors

A <u>unit vector</u> is a vector with length 1.

examples : $\vec{\imath}$, $-\vec{k}$, $\vec{\jmath}$, $\frac{1}{\sqrt{3}}(\vec{\imath}-\vec{\jmath}+\vec{k})$

Fact : Every nonzero vector is parallel to two unit vectors.

Picture:



check : $\left|\frac{1}{|\vec{a}|}\vec{a}\right| = \frac{1}{|\vec{a}|}|\vec{a}| = 1$ ...

example Find vector of length 7 with opposite direction of $\vec{a} = 3\vec{\imath}+4\vec{\jmath}$.

ans $|\vec{a}| = \sqrt{3^2+4^2} = 5$ so

$-\frac{1}{5}\vec{a} = -\frac{1}{5}(3\vec{\imath}+4\vec{\jmath})$ is unit vector in opp. direction.

$7\left(-\frac{1}{5}\vec{a}\right) = -\frac{7}{5}(3\vec{\imath}+4\vec{\jmath}) = \frac{-21}{5}\vec{\imath} - \frac{28}{5}\vec{\jmath}$

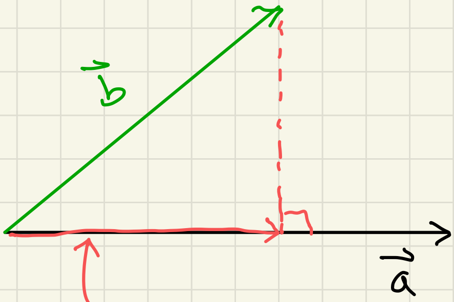
is desired vector.

Vector Projection:

(Stewart p. 851)

Scalar projection of **b** onto **a**: $\quad \text{comp}_{\mathbf{a}}\,\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$

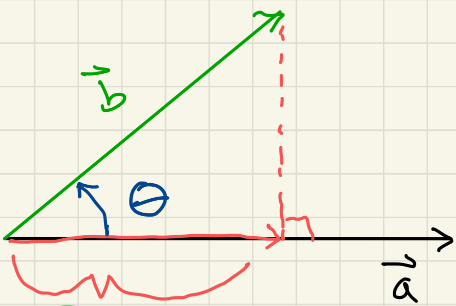
Vector projection of **b** onto **a**: $\quad \text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a}$



$\text{proj}_{\vec{a}}(\vec{b})$ = projection of $\vec{b}$ onto $\vec{a}$.

<u>note</u>: "scalar projection" is the length of "vector projection".

<u>Explain</u>



This length equals $\quad |\vec{b}|\cos\theta = \dfrac{|\vec{a}||\vec{b}|\cos\theta}{|\vec{a}|} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

The <u>cross product</u> of two vectors

$$\vec{a} = \langle a_1, a_2, a_3 \rangle \text{ and } \vec{b} = \langle b_1, b_2, b_3 \rangle$$

is the vector

$$\vec{a} \times \vec{b} = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle$$

<u>examples</u> $\vec{i} \times \vec{j} = \vec{k}$, $\langle 2,0,-3 \rangle \times \langle 0,4,1 \rangle = \langle 12,-2,8 \rangle$

<u>Why is this important?</u> If you carefully expand out

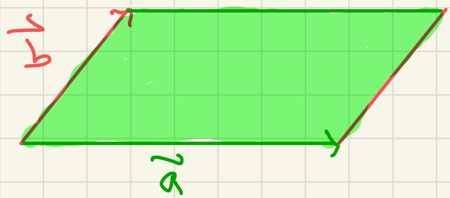
$$(\vec{a} \times \vec{b}) \cdot \vec{a} = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle \cdot \langle a_1, a_2, a_3 \rangle$$

you will get 0. (Try it.)

This means that the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$. It is also perpendicular to $\vec{b}$.

Another interesting fact is:

$|\vec{a} \times \vec{b}|$ equals the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$.



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11 Properties of the Cross Product If **a**, **b**, and **c** are vectors and c is a scalar, then

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

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14 The volume of the parallelepiped determined by the vectors **a**, **b**, and **c** is the magnitude of their scalar triple product:

$$V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$$