Chap 10 # 1(c), #7, #10

Chap 11 # 4, 5

#10 True or False

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

(a) $r\cos\theta = 5 \Rightarrow x = 5$ true

(b) $r = \cos\theta$ is a circle true

$r^2 = r \cdot r = r \cdot \cos\theta$

$x^2 + y^2 = x \Longrightarrow x^2 - x + \frac{1}{4} + y^2 = 0\ \frac{1}{4}$

$(x - 1/2)^2 + y^2 = \frac{1}{4}$

center: $(1/2, 0)$
radius: $1/2$

(c) $r = f(\theta)$ $\begin{cases} x = f(\theta)\cos\theta \\ y = f(\theta)\sin\theta \end{cases}$ true

(d) true

(h) $\begin{cases} x = t^2 - t \\ y = 3t^3 \end{cases}$ true goes thru $(0, 3)$? $t = 1$

(e) $r = \cos(4\theta)$

(f) $\begin{cases} x = 3\sin t \\ y = -3\cos t \end{cases}$ is a circle true

$x^2 + y^2 = 9\sin^2 t + 9\cos^2 t = 9$
$= 9(\sin^2 t + \cos^2 t)$

(g) $\begin{cases} x = t^2 - t \\ y = 3t^3 \end{cases}$ $\begin{cases} x = t - 1 \\ y = -2t + 1 \end{cases}$ true

$t = x + 1$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{9t^2}{2t - 1} ? = -2$

$y = -2(x+1) + 1$

$y = -2x - 1$

line w/ slope -2

$9t^2 = -4t + 2$

$9t^2 + 4t - 2 = 0$

Solve for t?

$b^2 - 4ac = 4^2 - (9)(-2) =$ positive

#7

$$C: \begin{cases} x = t^2 + 2t = t(t+2) \\ y = t^3 - t \end{cases} \qquad -\infty < t < \infty$$

y-intercepts: $x = 0$

$t = 0$ or $t = -2$

$x'(t) = dx/dt = 2t + 2$ 'critical #'s for $x(t)$: $t = -1$

$y'(t) = dy/dt = 3t^2 - 1$ critical #'s for $y(t)$: $t = \pm 1/\sqrt{3}$

horizontal motion

$x' < 0$ | -1 | $x' > 0$ (t-axis)

$\leftarrow$ | | $\rightarrow$

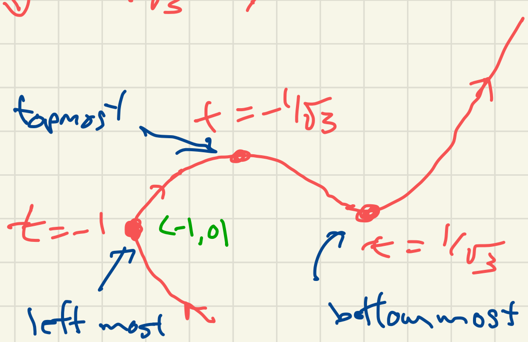
vertical motion

$y' > 0$ | $-\frac{1}{\sqrt{3}}$ | $y' < 0$ | $1/\sqrt{3}$ | $y' > 0$ (t-axis)

$\uparrow$ | | $\downarrow$ | | $\uparrow$

$\nwarrow$ | -1 | $\nearrow$ | $-1/\sqrt{3}$ | $\searrow$ | $1/\sqrt{3}$ | $\nearrow$

t	(x,y)
-1	$(-1,0)$
$-1/\sqrt{3}$	
$1/\sqrt{3}$	



vertical tangents $t = -1$

$$C: \begin{cases} x = t^2 + 2t \\ y = t^3 - t \end{cases}$$

$$x'(t) = \frac{dx}{dt} = 2t + 2$$

$$y'(t) = \frac{dy}{dt} = 3t^2 - 1$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2-1}{2t+2}$$

d) $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{3t^2-1}{2t+2}\right]$$

$$= \frac{\frac{d}{dt}\left[\frac{3t^2-1}{2t+2}\right]}{dx/dt} = \frac{1}{2t+2}\left(\frac{6t(2t+2)-(3t^2-1)(2)}{(2t+2)^2}\right)$$

$$= \frac{1}{dx/dt} \cdot \frac{d}{dt}\left[\frac{3t^2-1}{2t+2}\right]$$

$$\cos(\pi/3) = 1/2$$

when $t=-1$ $$\left.\frac{d^2y}{dx^2}\right|_{t=-1} = \frac{-4}{0}$$

chapter 11 Review

#4 Geometric Series

r is a constant

$$1 + r + r^2 + r^3 + r^4 + \cdots = \sum_{n=1}^{\infty} r^{n-1} = \sum_{n=0}^{\infty} r^n$$

- This converges and has sum $\frac{1}{1-r}$ when $|r|<1$.
- This diverges when $|r| \geq 1$.

(a) $$\sum_{n=0}^{\infty} \frac{3^n}{4^n} = \sum_{n=0}^{\infty} (3/4)^n$$ ← take $r = 3/4$

converges

$$= \frac{1}{1-3/4} = \frac{1}{1/4} = 4$$

(b) $$\sum_{n=1}^{\infty} \frac{3^n}{4^n} = \Big(\sum_{n=0}^{\infty} \frac{3^n}{4^n}\Big) - \underbrace{\Big(\frac{3^0}{4^0}\Big)}_{n=0 \text{ term}} = 4-1 = 3$$

(a) $= 1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \cdots$

(b) $= \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \cdots$

(c) $$\sum_{n=0}^{\infty} \frac{3^{n-2}}{4^{n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{36} (3/4)^n = \frac{1}{36} \sum_{n=0}^{\infty} (3/4)^n$$

$$= 4/36 = 1/9$$

$$\frac{3^{n-2}}{4^{n+1}} = \frac{3^{-2} \cdot 3^n}{4 \cdot 4^n} = \frac{3^{-2}}{4}\left(\frac{3}{4}\right)^n = \frac{1}{36}\left(\frac{3}{4}\right)^n$$

(d) $\sum_{n=0}^{\infty} \frac{3^{-n}}{4^{2n}} = \sum_{n=0}^{\infty} \left(\frac{1}{48}\right)^n = \frac{1}{1-\frac{1}{48}} = \frac{48}{47}$

$r = \frac{1}{48}$

$$\frac{3^{-n}}{4^{2n}} = \frac{\left(\frac{1}{3^n}\right)}{(4^2)^n} = \frac{1}{3^n}\,\frac{1}{16^n} = \frac{1}{48^n}$$

(e) $\sum_{n=0}^{\infty} \frac{3^{-n}}{4^{-2n}} = \sum_{n=0}^{\infty} \frac{4^{2n}}{3^n} = \sum_{n=0}^{\infty} \left(\frac{16}{3}\right)^n$

<u>diverges</u> $\quad r = \frac{16}{3} > 1$

<u>#5</u> $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = \ln(2)$

$\qquad\qquad\qquad ||$

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$

(a) $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \ln(2)$

(b) $\sum_{n=3}^{\infty} (-1)^{n+1} \frac{1}{n} = \left(\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}\right) - \Big(\underset{n=1}{1} - \underset{n=2}{\tfrac{1}{2}} \Big)$

$= \ln(2) - \frac{1}{2}$

(c) $\underline{\sum_{n=3}^{\infty} (-1)^n \frac{1}{n+1}} = -\frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \cdots$

$= \ln(2) - \left(1 - \frac{1}{2} + \frac{1}{3}\right) = \ln(2) - 5/6$

(d) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n} = 3 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 3 \ln(2)$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \leftarrow$$ If this written as $\sum_{n=5}^{\infty} a_n$ what would a_n be?

$$= \sum_{n=5}^{\infty} (-1)^{n-3} \frac{1}{n-4} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

(e) $\sum_{n=5}^{\infty} (-1)^{n-3} \frac{1}{n-4} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = \ln(2)$

$$= \sum_{n=5}^{\infty} (-1)^{n+1} \frac{1}{n-4}$$

b/c $(-1)^{n-3} = (-1)^{n+1} (-1)^{-4} = (-1)^{n+1}$