**Problem 1.** (10 points) Here is a purported proof that the integer 2 is smaller than the integer 1. Find any incorrect conclusions that are made in the “proof” and carefully explain what makes them invalid.

**Claimed Proof:** Let \( a \) and \( b \) be integers and assume that \( a < b \). It follows that \( a^2 < b^2 \) and this inequality can then be rewritten as
\[
0 < b^2 - a^2 = (b - a)(b + a).
\]
Dividing both sides of this inequality by the positive number \( b - a \) shows that \( b + a > 0 \). Subtracting \( a \) from both sides of the equation gives \( -a < b \). Now imagine choosing \( a = -2 \) and \( b = 1 \). Since \(-2 < 1\) the argument above shows that \(-(-2) = -a < b = 1\). Therefore \( 2 < 1 \).

**Problem 2.** (30 points) Give an element-wise proof of:

**Proposition:** For all sets \( A, B, \) and \( C \),
\[
A \cap (B \cup C) \subseteq (A \cap B) \cup C.
\]

**Problem 3.** (20 points) Let \( A, B, \) and \( C \) be sets.

(a) Give a counterexample showing that \( (A \cup B) - C \) need not be a subset of \( (A - B) \cup C \).

(b) If \( A, B, \) and \( C \) are sets where \( (A \cup B) - C \) is a subset of \( (A - B) \cup C \) what does that say about the relationship between the three sets?

**Problem 4.** (15 points) In this problem \( B \) and \( C \) are sets. Use an element-wise approach to prove the proposition that “If \( B \subseteq C \) then \( B - C = \emptyset \).”

**Problem 5.** (10 points) Consider the statement:

\[
\forall m \in \mathbb{N}, \exists n \in \mathbb{Z} \text{ such that } m^2 + n^2 \text{ is odd}
\]

(a) Write the statement as a concise English sentence.

(b) Write the negation of the statement as a concise English sentence.

(c) Verify that the statement is true.

**Problem 6.** (15 points) We say that an sg-path in the integer grid has property \( G \) provided that there are no two successive \( U \)’s in the RU-string associated with the path.

(a) List the RU-strings for all of the sg-paths from \((0,0)\) to \((2,2)\) that have property \( G \).

(b) There are 120 sg-paths from \((0,0)\) to \((3,7)\). How many of them have property \( G \)? Explain.

(c) There are 120 sg-paths from \((0,0)\) to \((7,3)\). How many of them pass through the grid point \((2,2)\) and have property \( G \)? Explain.

(d) How many of the sg-paths from \((0,0)\) to \((7,3)\) have property \( G \)?