2924 Problem Review Session
August 27, 2019

PROBLEM 1. Find an equation for the line tangent to the graph $y = 1 + \ln(5 - x^2)$ at the point $P = (2, 1)$. (Did you need to check that P is actually on the graph?)

PROBLEM 2. Determine whether or not the given function has an inverse function. Find an equation for the inverse if it has one. (a) $g(x) = \ln(x^2 + 1)$ (b) $f(x) = e^{-x^3}$

PROBLEM 3. The second derivative of a function $f(x)$ has equation $f''(x) = 4x^2(x - 300)$. Explain why the function has only one point of inflection even though $f''(x) = 0$ has two distinct solutions.

PROBLEM 4. Do the graphs of $y = e^x$ and $y = \ln(x)$ intersect? Justify your answer. (Suggestion: Do either of the curves intersect the line $y = x$?)

PROBLEM 5. Consider the function $h(x) = 10 \ln(x)$.
(a) Show that h has an inverse function h^{-1} and find an equation for it.
(b) Does the graph of $y = h(x)$ intersect the line $y = x$?
(c) Do the graphs of $y = h(x)$ and $y = h^{-1}(x)$ intersect?

PROBLEM 6. Find the area of the following regions and explain why they are equal.
(a) The region bounded by $y = 2$ (top), $y = e^x$ and the y-axis.
(b) The region below the graph of $y = \ln(x)$, above the x-axis and to the left of the vertical line $x = 2$.

PROBLEM 7. What is the domain of each of the following functions? Determine the intervals of increase/decrease and concavity for each function and sketch a rough graph.
(a) $f(x) = \ln(5 - x^2)$ (b) $g(x) = \exp(5 - x^2)$

PROBLEM 8. Evaluate the integrals:
(a) $\displaystyle\int \frac{x + 2}{4x^2 + 16}\, dx$ (b) $\displaystyle\int \frac{\sin(\ln(x)) \cos(\ln(x))}{x}\, dx$

PROBLEM 9. Find a point P on the graph $y = e^x$ where the tangent line at P goes through the origin.

PROBLEM 10. Differentiate each of the following:
(a) $\ln(x)^2$ (b) $\sqrt{e^x}$ (c) $(e^x - 1)/(e^x + 1)$ (d) $(e^x - 1)/(e^{-x} + 1)$ (e) $\log_5(x^2 + 1)$

PROBLEM 11. Let m and b be constants and assume that $m > 0$. Let $f(x) = \ln(x) - mx - b$.
(a) Determine the intervals of monotonicity (that is increasing/decreasing) for $f(x)$ and show that $f(x)$ has an absolute maximum value.
(b) Find an inequality relating m and b that (precisely) describes when the equation $f(x) = 0$ will have at least one solution.
(c) When does a line $y = mx + b$ with positive slope m and y-intercept b intersect the graph $y = \ln(x)$ of the logarithm function?