Math 2924, Problem Set

November 19

1. Find the Maclaurin series expansion for $f(x) = \dfrac{x^3}{1+x^2}$ and determine its radius of convergence. (Suggestion: rather than starting to work out derivatives of $f(x)$ try relating the function to a geometric series.)

2. Find the Taylor series of $h(x) = 1/x$ centered at $a = 3$ in two different ways.

3. Let $P(x) = x^5 + 9x^4 + 33x^3 + 61x^2 + 57x + 21$. Find the Maclaurin series expansion for $P(x)$. What is its radius of convergence?

4. For the polynomial function $P(x)$ in the previous problem, determine the Taylor series expansion centered at $x = -2$. Use your answer to calculate the value of $P(-1.9)$ by hand.

5. Determine the limit L as x approaches 0 of $f(x) = (6e^x - 6 - 6x - 3x^2 - x^3)/x^4$ by first finding the Maclaurin series expansion of the numerator of $f(x)$.

6. Give the Taylor series expansions with indicated center a:

(a) $f(x) = x^2 e^{-3x}, \quad a = 0$

(b) $f(x) = \displaystyle\int \frac{\sin x}{x}\, dx, \quad a = 0$

(c) $f(x) = e^x, \quad a = 2$

7. What are Maclaurin series for $1/(1+x^2)$, $\arctan(x)$, $\arctan(-x)$ and $\arctan(2x^2)$?
Comment: There are a handful of functions whose Maclaurin series you should remember. The inverse tangent function is one.

8. The hyperbolic cosine function $\cosh(x)$ is an even function. How can you see that by looking at its Maclaurin series? Can you see that $\sinh(x)$ is an odd function from its Maclaurin series?

9. Find a few of the first terms of the Maclaurin series for $\tan(x)$. Can you see why finding a pattern for the general term of this series might be impossible?