Math 2924 Problem Session
12/3/19

PROBLEM 1. Consider the infinite series $\displaystyle\sum_{n=2}^{\infty} \frac{2}{5^{2n-3}n!}$.

(a) What is the third term of the series?

(b) What is the third partial sum of the series?

(c) Does the sequence of terms of the series converge? If so what is its limit as n goes to infinity?

(d) Does the sequence of partial sums of the series converge? If so what is its limit as n goes to infinity?

PROBLEM 2. Let $\mathbf{a}$ be the vector $\mathbf{a} = \langle 5, -3 \rangle = 5\mathbf{i} - 3\mathbf{j}$.

(a) Describe in algebraic form all vectors $\mathbf{b}$ parallel to $\mathbf{a}$.

(b) Describe all units vectors which are parallel to $\mathbf{a}$ in algebraic form.

(c) Describe in algbraic form all vectors with the same length as $\mathbf{a}$.

(d) Find two vectors perpendicular to $\mathbf{a}$.

(e) Describe in algebraic form all vectors perpendicular to $\mathbf{a}$.

PROBLEM 3. Let $\mathbf{a}$ and $\mathbf{b}$ be two non-zero vectors and let θ be the angle between these two vectors.

(a) If $\mathbf{a} \cdot \mathbf{a} = 0$ what does θ equal? Explain.

(b) Use (a) to describe all vectors which are perpendicular to $\mathbf{a} = \langle a_1, a_2 \rangle$ in algebraic form.

PROBLEM 4. Use dot products to determine the values of the cosines of the three angles in the triangle PQR where $P = (1, -3)$, $Q = (2, 5)$ and $R = (-3, -1)$. Are any of these angles obtuse? How can you use the dot product to determine when an angle is obtuse?

PROBLEM 5. Let T be a point $(0, y)$ on the y-axis, and let P and Q be the points in the previous problem. Find all values of y for which the triangle PQT is a right triangle. (hint: there are four of them.)

PROBLEM 6. Let P and Q be the points from problem 5.

Is there a point $S = (x, y)$ such that the triangle PQS is a right triangle? If so, describe all of the possible points S.

PROBLEM 7. Consider the infinite series $\sum_{k=1}^{\infty} \frac{3}{3k+4} - \frac{3}{3k+1}$.

(a) Write out the first three partial sums of this series. Then find a formula for the kth partial sum s_k.

(b) Find the sum of $\sum_{k=1}^{\infty} \frac{1}{9k^2+15k+4}$ by comparing with (a).