

**2924 Problem Review Session**  
**September 10, 2019**

PROBLEM 1. Use logarithmic differentiation to find  $dy/dx$  where  $y = x^2 e^{\sin(x)}$ . Compare your answer to what you would get using the product rule.

PROBLEM 2. Do the graphs of the exponential function  $f(x) = e^x$  and the logarithm function  $g(x) = \ln(x)$  have any common tangents?

- (a) Make a sketch of the two curves and use it to guess how many lines will be tangent to both curves.
- (b) Find the slope/intercept equations for any lines which are tangent to both curves.

PROBLEM 3. Find the limits:

- (a)  $\lim_{x \rightarrow \infty} x^2 \ln(x)$
- (b)  $\lim_{x \rightarrow e} x^2 \ln(x)$
- (c)  $\lim_{x \rightarrow 0^+} x^2 \ln(x)$
- (d)  $\lim_{x \rightarrow 0^-} x^2 \ln(x)$
- (e)  $\lim_{x \rightarrow 0} \frac{\sin(x) - \tan(x)}{x^3}$
- (f)  $\lim_{x \rightarrow 0} \frac{1}{\sin(x)} - \frac{1}{x}$
- (g)  $\lim_{x \rightarrow \infty} x^{(1/x)}$

PROBLEM 4. Use a calculus analysis to sketch the graph of  $f(x) = x^2 \ln(x)$ . (Compare with parts (a–d) of the previous problem.)

PROBLEM 5. Let  $G(x) = \arctan(\tan(x))$ .

- (a) Complete the description: "For a real number  $t$ ,  $\arctan(t)$  equals the angle ..."
- (b) Determine the values  $G(0)$ ,  $G(\pi/4)$ ,  $G(\pi)$ ,  $G(3\pi/4)$ ,  $G(-3\pi/4)$ .
- (c) Find the limits  $\lim_{x \rightarrow \pi/2^-} G(x)$  and  $\lim_{x \rightarrow \pi/2^+}$ .
- (d) Explain that  $G(x)$  is an odd function and also a periodic function. (Write equations for both.)
- (e) What is the domain of this function?
- (f) Sketch the graph of  $G(x)$ .