

**2924 Problem Set #6**  
**October 8, 2019**

PROBLEM 1. Verify that  $\cosh(2x) = 2 \cosh^2(x) - 1$  and  $\sinh(2x) = 2 \sinh(x) \cosh(x)$ . Then determine  $\int \cosh^2(x) dx$  and  $\int \cosh^3(x) dx$ .

PROBLEM 2. (a) Sketch the curves  $y = \sinh(x)$  and  $y = \cosh(x)$ .

(b) Does the region in the first quadrant between these two curves have finite area?

PROBLEM 3. The point  $(4, 3)$  is a point of self-intersection of the curve  $C : x = 3t^2 + 1, y = 2t^3 + 1$ . Determine equations for the different lines that are tangent to  $C$  at that point.

PROBLEM 4. Determine a good viewing window for the parametric curve  $C : x = t^4 + 4t^3 - 8t^2, y = 2t^2 - t$ . Ideally, a good viewing window should contain all points where the motion describing the curve changes either horizontal or vertical direction, as well as the  $x$ - and  $y$ -intercepts. It might also include points of inflection for  $x(t)$  and  $y(t)$ .

PROBLEM 5. Consider the curve  $C$  described by an object in motion with parametric equations

$$C : \begin{cases} x &= t^2 \\ y &= \cos(\pi t) \end{cases}$$

(a) Find all of the  $x$ - and  $y$ -intercepts of  $C$ .

(b) How do you know that  $C$  does not contain any points inside quadrant II or III?

(c) Locate the  $t$  intervals on which the object is moving left/right.

(d) Locate the  $t$  intervals on which the object is moving down/up.

(e) Sketch the curve  $C$  and briefly describe how the object moves along it.

PROBLEM 6. An object moves in the  $xy$ -plane according to the parametric equations  $x = x(t), y = y(t)$ . If both functions  $x(t)$  and  $y(t)$  are even functions what does that say about the motion? (Hint: The motion described in the first problem satisfied this condition.)

PROBLEM 7. In each of the following situations, a curve  $C$  with parametric equations  $x = x(t), y = y(t)$  (defined for all real parameters  $t$ ) will satisfy a symmetry property. What is it?

(a)  $x(t)$  is an even function and  $y(t)$  is an odd function.

(b)  $x(t)$  is an odd function and  $y(t)$  is an even function.

(c)  $x(t)$  is an odd function and  $y(t)$  is an odd function.