Math 2924–Calculus II
some Exam 1 Review Problems

PROBLEM 1. *(15 points)* Use a right triangle analysis to find
(a) $\tan\left(\cos^{-1}(\sqrt{7}/4)\right)$ (b) $\sec\left(\cos^{-1}(\sqrt{7}/4)\right)$

PROBLEM 2. *(15 points)* Let $f(x) = \dfrac{3x+2}{x-7}$. Find a formula for $f^{-1}(x)$. Briefly explain how you know f is a one-to-one function.

PROBLEM 3. *(25 points)* Consider the function $f(x) = 2\ln(x) + 1/x$.
(a) Describe the domain of this function and determine its first two derivatives.
(b) Determine the intervals of increase/decrease for $f(x)$, and indicate local extremes.
(c) Determine the intervals of concavity for $f(x)$, and indicate points of inflection.
(d) Identify the range of f and all $x-$ and $y-$intercepts.
(e) Use (a)–(d) to sketch a graph of $y = f(x)$. (NOTE: it may be helpful to know that $\lim_{x\to 0+} f(x) = \infty$.)

PROBLEM 4. *(15 points)* Differentiate (and simplify):
(a) $F(x) = \arctan(x^4)$, (b) $G(x) = \ln(\sec(x) + \tan(x))$ (c) $H(x) = x^{x^2}$

PROBLEM 5. *(15 points)* Suppose that $h(t) = Ce^{kt}$ for some constants C and k and that $h(10) = 100$ and $h'(10) = 100$. (a) Give a precise formula for $h(t)$. (b) If $h(t_1) = 1000$ what does t_1 equal?

PROBLEM 6. *(15 points)* One of the two integrals

$$\int \frac{e^x + \sin(x)}{e^x + \cos(x)}\,dx \qquad \text{and} \qquad \int \frac{e^x + \cos(x)}{e^x + \sin(x)}\,dx$$

is easy to compute and the other is very difficult. Identify the easier one and work it out.

PROBLEM 7. *(5 points)-bonus* Let f be a one-to-one function whose graph contains the point $P = (5, 2)$. If the tangent line to the graph of $y = f(x)$ at $(5, 2)$ is $y = -2x + 12$ what is tangent line to the graph of f^{-1} at the point $(2, 5)$ in slope/intercept form?

PROBLEM 8. Verify that each of the following is correct:
a) $\displaystyle\int_{1/\sqrt{2}}^{1} \frac{1}{\sqrt{1-x^2}}\,dx = \pi/4$
b) $\displaystyle\int \frac{\sin(\arctan(x))}{1+x^2}\,dx = \frac{-1}{\sqrt{1+x^2}} + C$

PROBLEM 9. Let $f(x) = \ln(2x^3 - 1)$.
a) Determine the domain of f.
b) Explain why f is a one-to-one function.
c) Find a formula for the inverse function $f^{-1}(x)$.

PROBLEM 10. Sketch the graph of the function $f(x) = x^3e^{-x}$. First determine the intervals of increasing/decreasing and the intervals of concavity for f, and clearly indicate any local extremes, points of inflection and asymptotes. (It may help to know that $\lim_{x\to\infty} f(x) = 0$.)

PROBLEM 11. Find the value of C for which the area of the region in the first quadrant under $y = e^x$ between $x = C$ and $x = C + 1$ is e^{100}. Is $\log_{10}(C)$ bigger than 2 or smaller than 2?

PROBLEM 12. (a) What are the domain and range of the function $f(x) = \tan^{-1}(x)$? Sketch the graph of this function.
(b) What is the domain of $g(x) = \ln(\tan^{-1}(x))$?

Problem 13. (a) What is the domain and range of the function $f(x) = \sin^{-1}(x)$? Sketch the graph of this function.
(b) What is the domain of $g(x) = \ln(\sin^{-1}(x))$?

Problem 14. If $f(x) = \dfrac{4x-1}{2x+3}$ find a formula for the inverse function $f^{-1}(x)$.

Problem 15. Consider the function $f(x) = \ln\ln\ln\ln(x)$ (that is $f(x) = \ln(\ln(\ln(\ln(x))))$.
a) Is $f(x)$ defined for $x = e$? Determine the domain of f.
b) Show that f is a strictly increasing function.
c) Find a formula for the inverse function $f^{-1}(x)$ (note that f has an inverse since it is one-to-one by (b).)

Problem 16. Work the integrals:
(a) $\displaystyle\int \sec^2(x)e^{\tan(x)}\,dx$
(b) $\displaystyle\int \frac{y}{y^2+1}\,dy$
(c) $\displaystyle\int_{1/2}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$
(d) $\displaystyle\int_{1}^{e} \frac{1}{x(1+(\ln(x))^2)}\,dt$

Problem 17. *(15 points)* Use a right triangle analysis to simplify each of:
(a) $\sin(\tan^{-1}(2/3))$ (b) $\cos(\tan^{-1}(2/3))$

Problem 18. Sketch the graphs of $y = e^x$ and $y = \ln(x)$ in the same picture.

Problem 19. Determine the intervals of monotonicity for the function $f(x) = \ln(x^2 - 2x + 2)$ and identify any local maximums or minimums for f. (Hint: The domain of f consists of all real numbers.)

Problem 20. If $f(x) = \dfrac{3-x}{5+2x}$ then find a formula for the inverse function $f^{-1}(x)$. Is $f(x)$ a one-to-one function?

Problem 21. Compute each of the following integrals:
(a) $\int e^{3x+1}\,dx$
(b) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$
(c) $\displaystyle\int_{1/e}^{e^2} \frac{1}{x}\,dx$
(d) $\displaystyle\int \frac{\sqrt{\ln(x)}}{x}\,dx$
(e) $\displaystyle\int \frac{1}{1+25x^2}\,dx$

Problem 22. Find the area of the region bounded by the hyperbola $y = 3/(x-2)$ and the three lines $y = 0$, $x = -1$ and x=-4. Sketch the region.

Problem 23. Compute each of the following and write your answer in simplest form.

(a) $\displaystyle\int_{1/\sqrt{e}}^{e^3} \frac{1}{x}\,dx$
(b) $\displaystyle\int e^{3x-2}\,dx$

(c) $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

(d) $\displaystyle\int \frac{(1+\ln(x))^{10}}{x}\,dx$

PROBLEM 24. Differentiate each of the following functions of x and write your answer in simplest form.
(a) $f_1(x) = e^{3x-2}$
(b) $f_2(x) = 5^{3x-2}$
(c) $f_3(x) = 2\ln(x^2+1) - 2x\arctan(x) + 1$

PROBLEM 25. Suppose that $f(x) = Ce^{-5x}$ for some constant C and that $f(0) = 320$. Find the value of x for which $f(x) = 10$. Write your answer in simplest form.

PROBLEM 26. Let $f(x) = -3x + 2$. (a) Find a formula for the inverse function $f^{-1}(x)$.
(b) In the same picture, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$. Clearly identify the coordinates of any points of intersection.

PROBLEM 27. Let $f(x) = (\ln(x))^2$.
(a) Describe the domain of f.
(b) Determine the first two derivatives of $f(x)$.
(c) Find the intervals on which f is increasing and decreasing and identify any local extremes.
(d) Find the intervals on which f is concave up and concave down.
(e) Give a rough sketch of the graph of $y = f(x)$ using the information in (a)-(d).