EXAM 4 – some review problems
Math 2924

PROBLEM 1. Describe what it mean for an infinite series $\sum_{n=1}^{\infty} a_n$ to converge conditionally? Then give (and briefly justify) an example of a series which converges conditionally.

PROBLEM 2. (a) Give an example of a power series that has infinite radius of convergence.
(b) If a power series $\sum_{n=0}^{\infty} c_n x^n$ has infinite radius of convergence what can be said about the radius of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{c_n}$? Explain your answer.

PROBLEM 3. Determine whether each of the following series converges absolutely, converges conditionally or diverges. Indicate any convergence test that you have used and clearly show that any necessary hypotheses for the test are met.

(a) $\displaystyle\sum_{n=0}^{\infty} \cos\left(\frac{3n+1}{2n^3-7}\right)$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{3n+1}{2n^3-7}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(n-1)}{n^2+3}$

PROBLEM 4. Give power series expansions with indicated center a for each function:
(a) $f(x) = e^{-x}, \quad a = 0$
(b) $f(x) = x^2 e^{-3x}, \quad a = 0$
(c) $\cosh(x) = (e^x + e^{-x})/2, \quad a = 0$
(d) $f(x) = \dfrac{1}{x}, \quad a = 2$
(e) $f(x) = \dfrac{1}{(1-x)^2}, \quad a = 0$

PROBLEM 5. Determine the radius of convergence and the interval of convergence for each of the power series:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n^2+1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{5^n(n^2+1)}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(x+3)^n}{5^n(n^2+1)}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2 x^n}{(2n)!}$

PROBLEM 6. Let $F(x) = 3x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$
(a) If $\sum_{n=0}^{\infty} c_n x^n$ is a power series representing $F(x)$ what do c_0, c_4 and c_7 equal?
(b) If $\sum_{n=0}^{\infty} c_n (x-2)^n$ is a power series centered at 2 representing $F(x)$ what do c_0, c_4 and c_7 equal?