

Sample EXAM 1
Math 2924

Name:

PROBLEM 1. (15 points) Find the derivatives of:

(a) $f(x) = e^{1/x}$

(b) $g(x) = e^{3x}/x^3$

(c) $h(x) = x^{\ln(x)}$

PROBLEM 2. (10 points) Explain briefly why $f(x) = x^5$ has an inverse function but $g(x) = x^4$ does not.

PROBLEM 3. (15 points) (a) Suppose $f(x) = 5(x - 1)^2 - 7$ where $x \geq 1$. Find a formula for $f^{-1}(x)$.
(b) Suppose $g(x) = 5(x - 1)^2 - 7$ where $x \leq 1$. Find a formula for $g^{-1}(x)$.

PROBLEM 4. (15 points) Let $f(t) = Ce^{kt}$ where C and k are constants. If $f'(0) = 1$ and $f'(2) = 100$ then what are the values of $f(0)$ and $f(1)$? (Be sure to simplify to the extent possible.)

PROBLEM 5. (25 points) Consider the function $f(x) = x (\ln(x))^2$.

a) Determine the domain of f and find its first two derivatives.

b) Determine the intervals of increase/decrease for f .

c) Determine the intervals of concavity for f .

d) Use the information from (a), (b) and (c) to sketch the graph of $y = f(x)$. (It may be helpful to know that $\lim_{x \rightarrow 0^+} f(x) = 0$.)

PROBLEM 6. (20 points) Compute the following integrals clearly showing any substitution used:

(a) $\int \frac{1}{1+x^2} dx$ (b) $\int_0^1 \frac{1}{1+x^2} dx$ (c) $\int \frac{1}{x\sqrt{\ln(x)}} dx$ (d) $\int_1^e \frac{1}{x\sqrt{\ln(x)}} dx$

(e) $\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx$ (f) $\int_{1/2}^{\sqrt{3}/2} \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx$