

Parametric Curves in Motion

example from class on October 2

first enter the functions :

```
In[ ]:= x[t_] := 6 - t^2
        y[t_] := -t^3/3 + 2t - 2
```

then the basic ParametricPlot command:

```
In[ ]:= ParametricPlot[{x[t], y[t]}, {t, -4, 3}]
```

add some modifiers (experiment with this---find many details through “?Plot” command) (Note: if the rendering of the curve seems choppy you can sometimes smooth it out by increasing PlotPoints, adding say “,PlotPoints->150” into the ParametricPlot command (or increase 150 as appropriate):

```
In[ ]:= ParametricPlot[{x[t], y[t]}, {t, -4, 3}
, Frame → True, Background → LightGreen, ImageSize → 300]
```

we can trace the curve by creating an animation. Note that this command is creating many frames as the value of b increments from -3.9 to 4 in steps of .05. (If the .05 is left off then usually Mathematica will make a good choice for the step size.)

(A technical point here is that the “Parametric Plot” command does not work on a t interval [-4,b] where b=-4, and this is why the starting value was chosen slightly larger than -4.) (after compiling click on the little plus symbol and a menu for the animation will appear):

```
In[ ]:= Manipulate[
ParametricPlot[{x[t], y[t]}, {t, -4, b}
, Frame → True, Background → LightGreen],
{b, -3.9, 4, .05}]
```

Notice that this animation is not coherent because Mathematica has chosen different viewing windows for each of the frames.

To fix this we'll set each viewing window as [-10,10]x[-10,10] using “PlotRange”. Also control the output size with “ImageSize”.

```
In[ ]:= Manipulate[
ParametricPlot[{x[t], y[t]}, {t, -4, b}
, Frame → True, Background → LightGreen,
PlotRange → {{-10, 10}, {-10, 10}}, ImageSize → 300],
{b, -3.9, 4, .05}]
```

Here's the basic command for graphing a point (with the viewing window has been set as [-10,10]x[-10,10]):

```
In[ ]:= Graphics[
  Point[{x[1], y[1]}]
  , Frame → True , Axes → True, Background → LightGreen,
  ImageSize → 200, PlotRange → {{-10, 10}, {-10, 10}}]
```

add some modifiers to go with the "Point" command:

```
In[ ]:= Graphics[
  {Blue, PointSize[Medium], Point[{x[1], y[1]}]}
  , Frame → True , Axes → True, Background → LightGreen,
  ImageSize → 200, PlotRange → {{-10, 10}, {-10, 10}}]
```

Now we can put the object (point) in motion

```
In[ ]:= Manipulate[
  Graphics[
    {Blue, PointSize[Medium], Point[{x[b], y[b]}]}
    , Frame → True, Axes → True, Background → LightGreen,
    ImageSize → 200, PlotRange → {{-10, 10}, {-10, 10}}]
  ,
  {b,
   -4,
   4}]
```

To have it draw the moving curve with the moving point, the "Show" command is needed to combine both "Graphics" and "ParametricPlot" in one picture. (You can drag the picture to make it bigger, or reset ImageSize larger.):

```
In[ ]:= Manipulate[
  Show[
    {
      Graphics[
        {Blue, PointSize[Medium], Point[{x[b], y[b]}]}
        , Frame → True , Axes → True, Background → LightGreen,
        ImageSize → 200, PlotRange → {{-10, 10}, {-10, 10}}]
      , ParametricPlot[{x[t], y[t]}, {t, -3.99, b}]
    }
  ], {b, -4, 4}]
```

similar treatment for a different curve: the epitrochoid, problem #55 page 696

```
In[ ]:= x[t_] := 11 Cos[t] - 4 Cos[11 t/2]
        y[t_] := 11 Sin[t] - 4 Sin[11 t/2]
```

```
In[ ]:= b = 1;
        ParametricPlot[{x[t], y[t]}, {t, -3, b}
  , Frame → True, Background → LightGreen]
```

```

In[ ]:= Graphics[
  Point[{x[1], y[1]}]
  , Frame → True ]

In[ ]:= Graphics[
  {Blue, PointSize[Large], Point[{x[1], y[1]}]}]
  , Frame → True]

In[ ]:= Show[
  {
    ParametricPlot[{x[t], y[t]}, {t, 0, 1}, Frame → True, Background → LightGreen],
    Graphics[{Blue, PointSize[Large], Point[{x[1], y[1]}]}]
  }
]

In[ ]:= Manipulate[
  Show[
    {ParametricPlot[{x[t], y[t]}, {t, 0, b}, Background → LightGreen],
    Graphics[{Blue, PointSize[Large], Point[{x[b], y[b]}]}]}
  ]
  , {b, .01, 1}]

In[ ]:= Manipulate[
  Show[
    {ParametricPlot[{x[t], y[t]}, {t, -3, b}, Background → LightGreen
    , PlotRange → {{-15, 15}, {-15, 15}}],
    Graphics[{Blue, PointSize[Large], Point[{x[b], y[b]}]}]}
  ]
  , {b, .01, 6}]

In[ ]:= Manipulate[
  Show[
    {ParametricPlot[{x[t], y[t]}, {t, 0, b}, Background → LightGreen
    , PlotRange → {{-15, 15}, {-15, 15}}],
    Graphics[{Blue, PointSize[Large], Point[{x[b], y[b]}]}]}
  ]
  , {b, .01, 4 * Pi}]

```