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SURVEY ARTICLE: CHARACTERIZATIONS OF THE SAITO-KUROKAWA LIFTING

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ABSTRACT. There are a variety of characterizations of Saito-Kurokawa lifts from elliptic modular forms to Siegel modular forms of degree 2. In addition to giving a survey of known characterizations, we apply a recent result of Weissauer to provide a number of new and simpler characterizations of Saito-Kurokawa lifts.

1. Introduction. Consider a classical modular form f of weight k for the group $\mathrm{SL}_2(\mathbf{Z})$ with Fourier expansion $\sum_{n\geq 1} a_n q^n$. If f is a Hecke eigenform, one can define an L-function $L(s,f) = \sum_{n\geq 1} a_n n^{-s}$ so that its completion satisfies a functional equation with $s \mapsto k-s$. In this setting, Deligne [**5, 6**] proved the Ramanujan conjecture: namely, that the (suitably normalized) Satake parameters of f are unimodular.

In the setting of Siegel modular forms, the naive generalization of the Ramanujan conjecture above is false. In particular, Saito and Kurokawa independently constructed and computed Hecke eigenforms that had Satake parameters not on the unit circle, see [**14**]. It was later understood that these modular forms were in fact lifts from elliptic modular forms, and so the naive generalization of the Ramanujan conjecture was adjusted to say that if a Siegel modular form is not a Saito-Kurokawa lift then it has unimodular Satake parameters. Weissauer proved this conjecture in [**24**].

In this paper, we exploit this recent result to provide a collection of new characterizations of Saito-Kurokawa lifts. The new characterizations we present in Section 4 can be formulated as being determined by a single condition at a single prime. The tools used in this result, aside from the Ramanujan conjecture, are elementary. Before we present this result, we describe a number of characterizations in the literature [**4,**

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7–10, 16, 20]. A common feature of these earlier characterizations, unlike the new ones we present, is that they are determined by an infinite number of conditions.

The paper is organized as follows. We begin by giving some background on Siegel modular forms and their Hecke operators and fixing some notation. In the subsequent section we describe the Saito-Kurokawa lift and the characterizations of its image that are found in the literature. We conclude by presenting our new characterizations.

2. Siegel modular forms and Hecke operators.

Let the symplectic group of similitudes of genus 2 be defined by

$$\mathrm{GSp}\,(4) := \{g \in \mathrm{GL}\,(4) : {}^t gJg = \lambda(g)J, \lambda(g) \in \mathrm{GL}\,(1)\} \quad \text{where } J = \begin{bmatrix} 0 & I_2 \\ -I_2 & 0 \end{bmatrix}.$$

Let $\mathrm{Sp}\,(4)$ be the subgroup with $\lambda(g) = 1$. The group $\mathrm{GSp}^+(4, \mathbf{R}) := \{g \in \mathrm{GSp}\,(4, \mathbf{R}) : \lambda(g) > 0\}$ acts on the Siegel upper half space $\mathcal{H}_2 := \{Z \in M_2(\mathbf{C}) : {}^t Z = Z, \mathrm{Im}\,(Z) > 0\}$ by

$$g\langle Z\rangle := (AZ + B)(CZ + D)^{-1}, \quad \text{where } g = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \in \mathrm{GSp}^+(4, \mathbf{R}), \quad Z \in \mathcal{H}_2. \tag{2.1}$$

Let $S_k^{(2)}$ be the space of holomorphic Siegel cusp forms of weight k, genus 2 with respect to $\Gamma^{(2)} := \mathrm{Sp}(4, \mathbf{Z})$. Then $F \in S_k^{(2)}$ satisfies

$$F(\gamma\langle Z\rangle) = \det\,(CZ + D)^k F(Z)$$

for all $\gamma = \left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right] \in \Gamma^{(2)}$ and $Z \in \mathcal{H}_2$.

For $M \in \mathrm{GSp}^+(4, \mathbf{R}) \cap M_4(\mathbf{Z})$, define the Hecke operator $T_k(\Gamma^{(2)} M \Gamma^{(2)})$ on $S_k^{(2)}$ as in [**1**, (1.3.3)]. For a positive integer m, we define the Hecke operator $T_k(m)$ by

$$T_k(m) := \sum_{\lambda(M)=m} T_k(\Gamma^{(2)} M \Gamma^{(2)}). \tag{2.2}$$

Let us assume that $F \in S_k^{(2)}$ is a Hecke eigenfunction with $T_k(m)F = \mu_F(m)F$. Using the generating function for $\mu_F(p^r)$ as in [**1**, Theorem 1.3.2] we see that there are two complex numbers α_p, β_p such that:

$$\mu_F(p) = p^{k-3/2}\big(\alpha_p + \alpha_p^{-1} + \beta_p + \beta_p^{-1}\big), \tag{2.3}$$

$$\mu_F(p^2) = p^{2k-3}\bigg((\alpha_p + \alpha_p^{-1})^2 + (\alpha_p + \alpha_p^{-1})(\beta_p + \beta_p^{-1}) + (\beta_p + \beta_p^{-1})^2 - 2 - \frac{1}{p}\bigg). \tag{2.4}$$

The numbers α_p and β_p are called the Satake parameters of F.

3. Saito-Kurokawa lifts and their characterization. For even k, we have the following diagram of lifts.

$$\begin{array}{ccc} J_{k,1}^{\text{cusp}} & \xrightarrow{\quad Ma \quad} & S_k^{(2)} \\ {\scriptstyle EZ}\uparrow & & \uparrow{\scriptstyle SK} \\ S_{k-(1/2)}^{+}(\Gamma_0(4)) & \xleftarrow{\quad Sh \quad} & S_{2k-2}(\mathrm{SL}_2(\mathbf{Z})). \end{array}$$

The four spaces in that diagram are:

- $S_{2k-2}(\mathrm{SL}_2(\mathbf{Z}))$ is the space of elliptic cusp forms of weight $2k-2$ with respect to $\mathrm{SL}_2(\mathbf{Z})$,
- $S_{k-(1/2)}^{+}(\Gamma_0(4))$ is the space of holomorphic cusp forms of weight $k - 1/2$ with respect to $\Gamma_0(4)$ in the Kohnen plus space,
- $J_{k,1}^{\text{cusp}}$ is the space of holomorphic cuspidal Jacobi forms of weight k and index 1, and
- $S_k^{(2)}$ is the space of holomorphic Siegel cusp forms of weight k, genus 2 with respect to $\Gamma^{(2)} := \mathrm{Sp}\,(4, \mathbf{Z})$.

The map Sh is the Shimura lift given explicitly by Kohnen [**12**], the map EZ is the lift from half integral weight modular forms to Jacobi forms given by Eichler and Zagier [**7**] and the map Ma is the lift from Jacobi forms to Siegel modular forms due to Maass [**15**]. Finally, the map SK is the Saito-Kurokawa lift given as the composition of the three lifts described above. If $f \in S_{2k-2}(\mathrm{SL}_2(\mathbf{Z}))$, then we write $F = SK(f)$ for the Saito-Kurokawa lift of f.

There are several different ways to characterize when $F \in S_k^{(2)}$ is in the image of the Saito-Kurokawa lift. Here, we present a few of these. We begin by describing a number of characterizations related to our new characterization found below.

3.1. The Maass space. Given a Siegel modular form F of degree 2, we ask if the form is a lift. The first characterization that allows us to answer this question is in terms of its Fourier coefficients. Let

$$F\left(\begin{bmatrix} \tau & z \\ z & \tau' \end{bmatrix}\right) = \sum_{n,r,m} A(n,r,m) e^{2\pi i(n\tau + rz + m\tau')}, \quad \begin{bmatrix} \tau & z \\ z & \tau' \end{bmatrix} \in \mathcal{H}_2, \tag{3.1}$$

be the Fourier expansion of F. The *Maass space* (also known as the Maass Spezialschar) consists of all F in $S_k^{(2)}$ satisfying

$$A(n,r,m) = \sum_{d|(n,r,m)} d^{k-1} A\left(\frac{nm}{d^2}, \frac{r}{d}, 1\right) \quad \text{for all } n, m, r \in \mathbf{Z}. \tag{3.2}$$

By [**2, 15, 26**], F is a Saito-Kurokawa lift if and only if F lies in the Maass space. A streamlined proof of this fact is contained in [**7**].

We remark that in [**9**] a different Spezialschar is defined. It is conjectured to be equal to the Maass space in general and they are proved to be the same in degree 2.

Both of these characterizations are fundamentally different than our new characterizations in that they are global characterizations. That is, they require knowing all the Fourier coefficients of F; in particular, they require checking infinitely many conditions.

We remark that Ikeda [**11**] has obtained a Saito-Kurokawa type lifting of elliptic cusp forms to Siegel modular forms of degree $2n$. The characterization of the image of the lifting via Maass conditions has been generalized to this higher rank situation in [**13, 25**].

3.2. The Maass p-space. Let p be a prime number. The *Maass p-space* consists of all F in $S_k^{(2)}$ satisfying

$$A(np,r,m)+p^{k-1}A\left(\frac{n}{p},\frac{r}{p},m\right) = p^{k-1}A\left(n,\frac{r}{p},\frac{m}{p}\right)+A(n,r,mp) \quad \text{for } n,r,m\in \mathbf{Z}. \tag{3.3}$$

Here, we understand that $A(\alpha,\beta,\gamma) = 0$ if one of α,β,γ is not an integer. If F is a Saito-Kurokawa lift, then it lies in the Maass p-space for every prime p; this follows by substituting (3.2) into (3.3). In [**20**], it was shown that if F lies in the Maass p-space for almost all p then F is a Saito-Kurokawa lift.

In [**10**] the condition that F is in the Maass p-space is "F satisfies p-Hecke duality" and in that paper the following is proved: suppose R is any set of prime numbers with Dirichlet density smaller than 1/8. Then F is a Saito-Kurokawa lift if F satisfies the p-Hecke duality for any p outside R. This can be thought of as an improvement on the result described above.

These two related characterizations, while, in a sense, local, also require checking infinitely many conditions: one condition for each prime in some infinite set.

Also in [**10**], the following result is proved. For a fixed positive even integer k, there exists a constant $c(k)$ depending only on the weight k such that a cusp form $F\in S_k^{(2)}$ is a Saito-Kurokawa lift if and only if F satisfies p-Hecke duality for all prime numbers $p\le c(k)$.

In [**22**], it is proved, under the assumption of the Ramanujan conjecture, that a Siegel modular form lies in the Maass space if and only if there exists a prime p such that F lies in the Maass p-space. This is an improvement of the previously described result. We recall here that Weissauer [**24**] has recently proved the Ramanujan conjecture in this context and so this result is now unconditional.

Both these results are local and require checking a condition for finitely many p. Nevertheless, they still each require checking infinitely many conditions as one iterates through the coefficients in (3.3). Our new characterization of Hecke eigenforms in the Maass space is an improvement on this result as it requires checking a single condition for a single prime. We identify the prime for which it should be checked, and it turns out the prime is independent of the weight k.

3.3. Other characterizations. Here we summarize a number of characterizations in the literature that are not as closely related to our characterizations found below. Each of them requires global information. In the first case, it is necessary to know all the coefficients of F; in the second case, it is necessary to be able to compute the pole of an L-function and so one needs to know all the Hecke eigenvalues of F; and, in the third, one needs to check infinitely many conditions, namely, that each Hecke eigenvalue is positive. The first characterization is not about eigenforms while the other two are.

3.3.1. A differential operator. In [**9**], a characterization of forms in the Maass space is given in terms of Taylor coefficients and a differential operator $\mathbf{D}_{k,2\nu} : M_k^{(2)} \to \mathrm{Sym}^2(M_{k+2\nu})$ where $v \in \mathbf{Z}_{\geq 0}$. The characterization is: a Siegel modular form F of degree 2 is in the Maass space if and only if $\mathbf{D}_{k,2\nu}F$ is in a "diagonal" subspace of $\mathrm{Sym}^2(M_{k+2\nu})$.

3.3.2. The L-function of a Saito-Kurokawa lift. Suppose F is a Hecke eigenform. Then [**8, 16**] show that F is a Saito-Kurokawa lift if and only if the spin L-function of F has poles. This result has been generalized in the following way. The Gritsenko lift is an analogue of the Saito-Kurokawa lift but yields a Siegel paramodular form of degree 2 and paramodular level N. In [**22**], it is proved that F is a Gritsenko lift if and only if its spin L-function has poles.

3.3.3. The Hecke eigenvalues of a Saito-Kurokawa lift. Suppose F is a Hecke eigenform. Up to now, the characterizations have been in terms of Fourier coefficients, poles of L-functions and Taylor series coefficients. The final characterization, due to Breulmann [**4**], is as follows: F is a Saito-Kurokawa lift if and only if $\mu_F(m) > 0$ for all positive integers m. This characterization is very succinct and appealing and has been studied further. In particular, the following refinement was obtained in [**19**, Corollary 3.2]: If F is not a Saito-Kurokawa lift, then there exists an infinite set S_F of prime numbers

p such that, if $p \in S_F$, then there exist infinitely many r such that $\mu_F(p^r) > 0$, and infinitely many r such that $\mu_F(p^r) < 0$.

4. New characterizations of Saito-Kurokawa lifts. The characterizations of Saito-Kurokawa lifts we have described above are, in a sense, not very effective. Given a Siegel modular form F, it is difficult to effectively determine if it is a Saito-Kurokawa lift.

Our new characterizations are local and can be used to develop a test that reduces the problem of determining if a form is a Saito-Kurokawa lift to checking a single local condition (in fact, the prime at which the condition should be checked is identified). In this sense, we feel that our characterizations are optimal.

The key points are the following estimates for the Satake parameters α_p, β_p of a cuspidal Siegel eigenform $F \in S_k^{(2)}$.

Ramanujan estimate [**24**]. *If F is not a Saito-Kurokawa lift, then for every prime p, we have*

$$|\alpha_p| = |\beta_p| = 1. \tag{4.1}$$

Saito-Kurokawa estimate. *In case F is a Saito-Kurokawa lift, then we have for every prime p, after possibly changing notations,*

$$\alpha_p = p^{1/2}, \qquad |\beta_p| = 1. \tag{4.2}$$

We get the above condition from the following relation between L-functions (see [**7**]):

$$L(s, SK(f)) = L(s, f)\zeta(s + 1/2)\zeta(s - 1/2). \tag{4.3}$$

The key point to note is that, if $F \in S_k^{(2)}$ is a Hecke eigenform, then either F satisfies the Ramanujan estimates (4.1) or it is a Saito-Kurokawa lift.

The following theorem implies that a Hecke eigenform $F \in S_k^{(2)}$ is a Saito-Kurokawa lift if and only if $\mu_F(37) > 4 \cdot 37^{k-3/2}$. However, we state it in a more general way.

Theorem 4.1. *Let $F \in S_k^{(2)}$ be a Hecke eigenform with eigenvalues $\mu_F(m)$ for any positive integer m. The following statements are equivalent.*

i) *F is a Saito-Kurokawa lift.*

ii) *There exists a prime p such that $\mu_F(p) > 4p^{k-3/2}$.*

iii) *For every prime $p \geq 37$ we have $\mu_F(p) > 4p^{k-3/2}$.*

iv) *There exists a prime p such that $\mu_F(p^2) > 10p^{2k-3}$.*

v) *For every prime $p \geq 17$ we have $\mu_F(p^2) > 10p^{2k-3}$.*

vi) *For every prime p, we have*

$$\mu_F(p^2) = \mu_F(p)^2 - (p^{k-1} + p^{k-2})\mu_F(p) + p^{2k-2}. \tag{4.4}$$

vii) *There exists a prime p such that*

$$\mu_F(p^2) = \mu_F(p)^2 - (p^{k-1} + p^{k-2})\mu_F(p) + p^{2k-2}. \tag{4.5}$$

Proof. If F is not a Saito-Kurokawa lift then, by (2.3), (2.4) and (4.1), we have $|\mu_F(p)| \leq 4p^{k-3/2}$ and $|\mu_F(p^2)| \leq 10p^{2k-3}$ for every prime p. If we substitute $\alpha_p = \sqrt{p}$ in (2.3) and (2.4), then we get $\mu_F(p) > 4p^{k-3/2}$ for $p \geq 37$ and $\mu_F(p^2) > 10p^{2k-3}$ for $p \geq 17$. This proves the equivalence of i)–v).

We will now show that i) $\Rightarrow$ vi) $\Rightarrow$ vii) $\Rightarrow$ i). Let us assume that F is a Saito-Kurokawa lift. Then, for any prime p, if we substitute $\alpha_p = \sqrt{p}$ in (2.3), solve for $\beta_p + \beta_p^{-1}$ and substitute the result in (2.4), we get (4.4). This gives i) $\Rightarrow$ vi). The implication vi) $\Rightarrow$ vii) is trivial. Now, suppose for some prime p the condition (4.5) is satisfied. Then

$$\begin{aligned} &\mu_F(p)^2 - \mu_F(p^2) - p^{2k-4} \\ &\quad = 2p^{2k-3} + p^{k-3/2}(p^{1/2} + p^{-1/2})\big(\mu_F(p) - p^{k-3/2}(p^{1/2} + p^{-1/2})\big). \end{aligned} \tag{4.6}$$

On the other hand, eliminating $\beta_p + \beta_p^{-1}$ from (2.3) and (2.4), we get

$$\begin{aligned} &\mu_F(p)^2 - \mu_F(p^2) - p^{2k-4} \\ &\quad = 2p^{2k-3} + p^{k-3/2}(\alpha_p + \alpha_p^{-1})\big(\mu_F(p) - p^{k-3/2}(\alpha_p + \alpha_p^{-1})\big). \end{aligned} \tag{4.7}$$

Hence, $Z_1 = p^{k-3/2}(p^{1/2} + p^{-1/2})$ and $Z_2 = p^{k-3/2}(\alpha_p + \alpha_p^{-1})$ are the roots of the same quadratic equation $Z^2 - \mu_F(p)Z + t = 0$, where the value of t is obtained from the above equations. If $Z_1 = Z_2$, then $\alpha_p = p^{\pm 1/2}$, and we obtain i). If $Z_1 \neq Z_2$, then, from $Z_1 + Z_2 = \mu_F(p)$, we see that $\beta_p = p^{\pm 1/2}$, and we obtain i). This completes the proof of the theorem. □

Remark 4.2. Weissauer's result holds for all Siegel modular forms of degree 2 that are not CAP representations. In the case of full level, the case to which Theorem 4.1 applies, by the work of Piatetski and Shapiro [**17**], Siegel modular forms of degree 2 that are CAP representations are precisely the images of the Saito-Kurokawa lifting considered above.

In the case of Siegel modular forms of degree 2 with level $N > 1$, the situation is considerably more complicated. For instance, as shown in [**23**], the same elliptic modular form can lift to two different Siegel modular forms, one with respect to the Siegel modular group $\Gamma_0(N)$ and the other with respect to the Siegel paramodular group $\Gamma^{\text{para}}(N)$. Therefore, to keep things simple, we have restricted ourselves to the full level case but point out that the informed and interested reader should be able to generalize our theorem to other situations using the same kind of arguments.

Finally, we give a characterization of non-Saito-Kurokawa lifts in terms of the growth of the sequence of numbers $|\mu_F(p^r)|$ as r goes to infinity. For a fixed prime p, the following condition was considered in [**18**]: For all $\epsilon > 0$, there exists a positive constant $C_\epsilon > 0$, depending only on p and ϵ, such that

$$|\mu_F(p^r)| \leq C_\varepsilon p^{r(k-3/2+\varepsilon)} \quad \text{for all } r \geq 0. \tag{4.8}$$

It was shown that this condition is equivalent to the Ramanujan estimate (4.1) for the Satake parameters at p. For the case of $\mathrm{Sp}\,(4, \mathbf{Z})$, using the Ramanujan estimate (4.1), condition (4.8) can be written in the more precise (but equivalent) form

$$|\mu_F(p^r)| \leq \left(\binom{r+3}{3} + p^{-1} \binom{r+1}{3} \right) p^{r(k-3/2)} \tag{4.9}$$

$$\leq \frac{3}{2} \binom{r+3}{3} p^{r(k-3/2)} \quad \text{for all } r \geq 0. \tag{4.10}$$

Since (4.1) for *one* p implies (4.1) for *all* p, we obtain the following result.

Theorem 4.3. *Let $F \in S_k^{(2)}$ be a Hecke eigenform with eigenvalues $\mu_F(m)$ for any positive integer m. The following statements are equivalent.*

i) *F is not a Saito-Kurokawa lift.*

ii) *There exists a prime p such that* (4.10) *holds.*

iii) *For every prime p* (4.10) *holds.*

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