MATH 3113

Midterm II, Form A

April 13, 2007

Name :

I.D. no. :

- Calculators are not allowed. The problems are set so that you should not need calculators at all.

- Show as much work as possible. Answers without explanation will not receive any credit.

- Best of Luck.

i) (20 Points)

a) Find $\mathcal{L}^{-1}\{\frac{1}{s^{7/3}}\}$ $= \mathcal{L}^{-1}\left\{\frac{1}{\Gamma(7/3)} \cdot \frac{\Gamma(7/3)}{s^{7/3}}\right\}$

$$= \frac{1}{\Gamma(7/3)} \mathcal{L}^{-1}\left\{\frac{\Gamma(7/3)}{s^{7/3}}\right\} = \frac{t^{4/3}}{\Gamma(7/3)}$$

b) Find $\mathcal{L}^{-1}\{\frac{36}{s-5}\}$ $= 36\, \mathcal{L}^{-1}\left\{\frac{1}{s-5}\right\} = 36\, e^{5t}$

c) Find $\mathcal{L}^{-1}\{\frac{3s+4}{s^2+4}\}$ $= 3\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} + 2\, \mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$

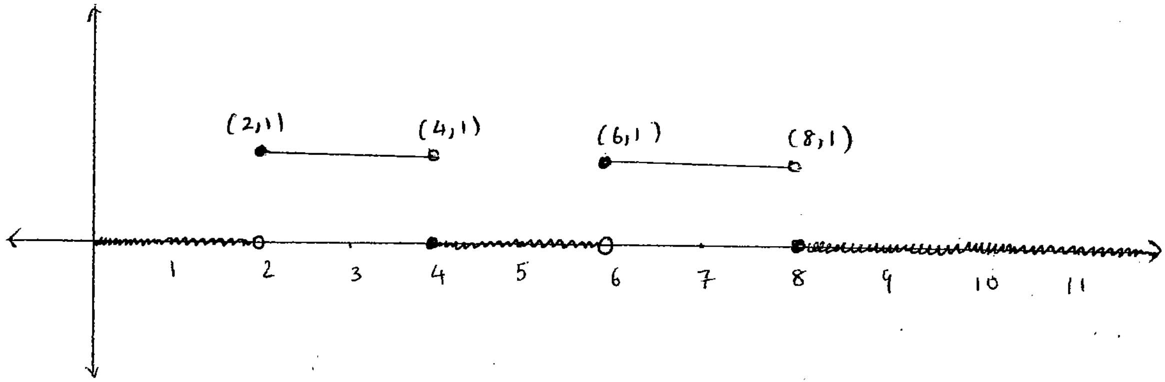
$$= 3\cos(2t) + 2\sin(2t)$$

d) Find $\mathcal{L}^{-1}\{\frac{2s+7}{9-s^2}\}$

$$= -2\, \mathcal{L}^{-1}\left\{\frac{s}{s^2-9}\right\} - \frac{7}{3}\, \mathcal{L}^{-1}\left\{\frac{3}{s^2-9}\right\}$$

$$= -2\cosh(3t) - \frac{7}{3}\sinh(3t)$$

ii) (20 Points) Find the Laplace Transform of the function $f(t)$ given by the following graph :



(I) $$f(t) = u(t-2) - u(t-4) + u(t-6) - u(t-8)$$

$$\Rightarrow \mathcal{L}\{f(t)\} = \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s} + \frac{e^{-6s}}{s} - \frac{e^{-8s}}{s}$$

<u>Alternatively</u> (II)

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \underbrace{\int_0^2 \cancel{e^{-st}} f(t)\,dt}_{0} + \int_2^4 e^{-st} f(t)\,dt + \underbrace{\int_4^6 e^{-st} \cancel{f(t)}\,dt}_{0} + \int_6^8 e^{-st} f(t)\,dt + \underbrace{\int_8^\infty \cancel{e^{-st}} f(t)\,dt}_{0}$$

$$= \int_2^4 e^{-st}\,dt + \int_6^8 e^{-st}\,dt = \left.\frac{e^{-st}}{-s}\right|_2^4 + \left.\frac{e^{-st}}{-s}\right|_6^8$$

$$= \left[\frac{e^{-4s}}{-s} + \frac{e^{-2s}}{s}\right] + \left[\frac{e^{-8s}}{-s} + \frac{e^{-6s}}{s}\right]$$

Note $f(t)$ is <u>not</u> a periodic function!!!

iii) (20 Points) Using Laplace Transforms solve the following Initial Value Problem :

$$x'' + 2x = 23\cos(5t), \qquad x(0) = x'(0) = 0.$$

$$s^2 X(s) + 2X(s) = \frac{23s}{s^2+25}$$

$$\Rightarrow X(s) = \frac{23s}{(s^2+2)(s^2+25)} = \frac{As+B}{s^2+2} + \frac{Cs+D}{s^2+25}$$

$$\Rightarrow 23s = (As+B)(s^2+25) + (Cs+D)(s^2+2)$$

$$= (A+C)s^3 + (B+D)s^2 + (25A+2C)s + (25B+2D)$$

$$\Rightarrow A+C=0, \quad B+D=0, \quad 25A+2C=23, \quad 25B+2D=0$$

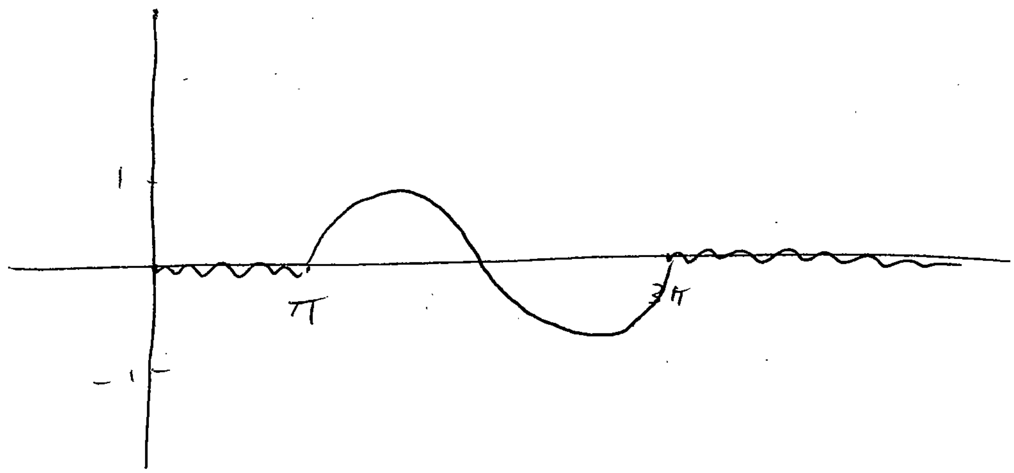
$$\Rightarrow \boxed{B=D=0, \quad A=1, \quad C=-1}$$

$$\Rightarrow X(s) = \frac{s}{s^2+2} - \frac{s}{s^2+25}.$$

$$\Rightarrow \boxed{x(t) = \cos(\sqrt{2}t) - \cos(5t)}$$

iv) (20 Points) Let $f(t) = u(t-\pi)\sin(t-\pi) - u(t-3\pi)\sin(t-3\pi)$.

a) Sketch the graph of $f(t)$.



b) Find the Laplace Transform of $f(t)$.

$$\mathcal{L}\{f(t)\} = e^{-\pi s}\frac{1}{s^2+1} - e^{-3\pi s}\frac{1}{s^2+1}$$

$$= \frac{e^{-\pi s} - e^{-3\pi s}}{s^2+1}$$

v) (15 Points) State whether the following statements are true or false. Show your answer by making a circle on TRUE or FALSE.

a) $\mathcal{L}^{-1}\{\ln(s+2)\} = -\frac{1}{t}e^{-2t}$ TRUE FALSE

b) $\mathcal{L}\{t\sin(t)\} = \frac{2s}{(s^2+1)^2}$ TRUE FALSE

c) $\mathcal{L}\{f(t).g(t)\} = \mathcal{L}\{f(t)\}.g(t) + f(t).\mathcal{L}\{g(t)\}$ TRUE FALSE

d) If $f(t)$ is a periodic function with period p and $g(t)$ is another function such that $g'(t) = f(t)$ then $g(t)$ is also periodic with period p. TRUE FALSE

e) Let $f(t) = 1$ and $g(t) = 1$ then the convolution product is

$(f * g)(t) = t$ TRUE FALSE

vi) (5 Points) Find $\mathcal{L}\{te^t\sin(2t)\}$.

$$\mathcal{L}\{te^t\sin(2t)\} = -\frac{d}{ds}\left[\mathcal{L}\{e^t\sin(2t)\}\right] = -\left[\frac{2}{(s-1)^2+4}\right]'$$

$$= \cancel{4}\cancel{2} \; -2\left[-\frac{2(s-1)}{[(s-1)^2+4]^2}\right] = \frac{4(s-1)}{[(s-1)^2+4]^2}$$