MATH 3113

Midterm II, Form B

April 13, 2007

Name :

I.D. no. :

- Calculators are not allowed. The problems are set so that you should not need calculators at all.

- Show as much work as possible. Answers without explanation will not receive any credit.

- Best of Luck.

i) (20 Points)

a) Find $\mathcal{L}^{-1}\{\frac{1}{s^{9/4}}\}$ $= \mathcal{L}^{-1}\left\{\frac{1}{\Gamma(9/4)} \cdot \frac{\Gamma(9/4)}{s^{9/4}}\right\}$

$= \frac{1}{\Gamma(9/4)} \mathcal{L}^{-1}\left\{\frac{\Gamma(9/4)}{s^{9/4}}\right\} = \frac{1}{\Gamma(9/4)} t^{5/4}$

b) Find $\mathcal{L}^{-1}\{\frac{23}{s-7}\}$

$= 23\, \mathcal{L}^{-1}\left\{\frac{1}{s-7}\right\} = 23 e^{7t}$

c) Find $\mathcal{L}^{-1}\{\frac{5s+9}{s^2+9}\}$

$= 5\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} + 3\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$

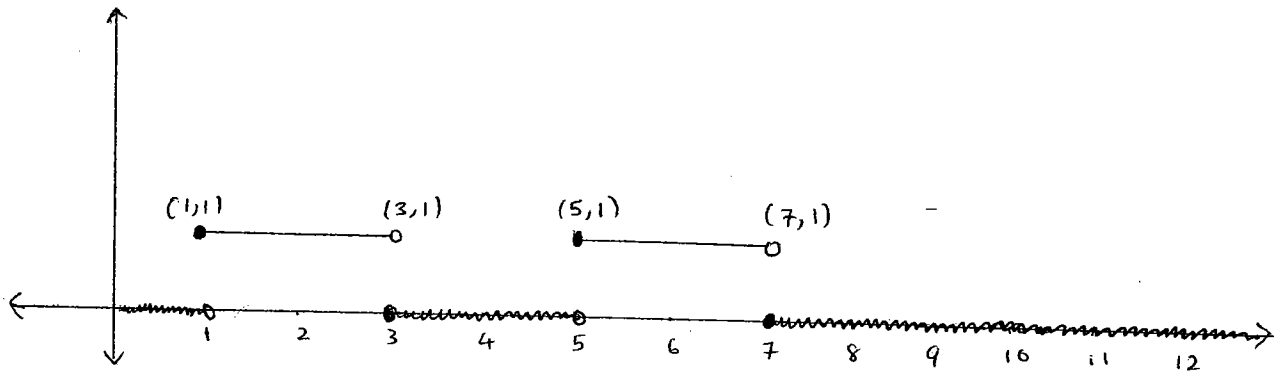
$= 5\cos(3t) + 3\sin(3t)$

d) Find $\mathcal{L}^{-1}\{\frac{3s+7}{4-s^2}\}$

$= -3\mathcal{L}^{-1}\left\{\frac{s}{s^2-4}\right\} - \frac{7}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2-4}\right\}$

$= -3\cosh(2t) - \frac{7}{2}\sinh(2t)$

ii) (20 Points) Find the Laplace Transform of the function $f(t)$ given by the following graph :



$$f(t) = u(t-1) - u(t-3) + u(t-5) - u(t-7)$$

$$\Rightarrow \mathcal{L}\{f(t)\} = \frac{e^{-s}}{s} - \frac{e^{-3s}}{s} + \frac{e^{-5s}}{s} - \frac{e^{-7s}}{s}$$

Alternatively :

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \int_0^1 e^{-st} f(t)\,dt + \int_1^3 e^{-st} f(t)\,dt + \int_3^5 e^{-st} f(t)\,dt$$

$$+ \int_5^7 e^{-st} f(t)\,dt + \int_7^\infty e^{-st} f(t)\,dt$$

$$= \int_1^3 e^{-st}\,dt + \int_5^7 e^{-st}\,dt = \frac{e^{-st}}{-s}\Big|_1^3 + \frac{e^{-st}}{-s}\Big|_5^7$$

$$= \left[\frac{e^{-3s}}{-s} + \frac{e^{-s}}{s}\right] + \left[\frac{e^{-7s}}{-s} + \frac{e^{-5s}}{s}\right]$$

iii) (20 Points) Using Laplace Transforms solve the following Initial Value Problem :

$$x'' + 5x = \cos(2t), \qquad x(0) = x'(0) = 0.$$

$$s^2 X(s) + 5X(s) = \frac{s}{s^2+4}$$

$$\Rightarrow X(s) = \frac{s}{(s^2+5)(s^2+4)} = \frac{As+B}{s^2+5} + \frac{Cs+D}{s^2+4}$$

$$\Rightarrow s = (As+B)(s^2+4) + (Cs+D)(s^2+5)$$

$$= (A+C)s^3 + (B+D)s^2 + (4A+5C)s + (4B+5D)$$

$$\Rightarrow A+C=0, \quad B+D=0, \quad 4A+5C=1, \quad 4B+5D=0$$

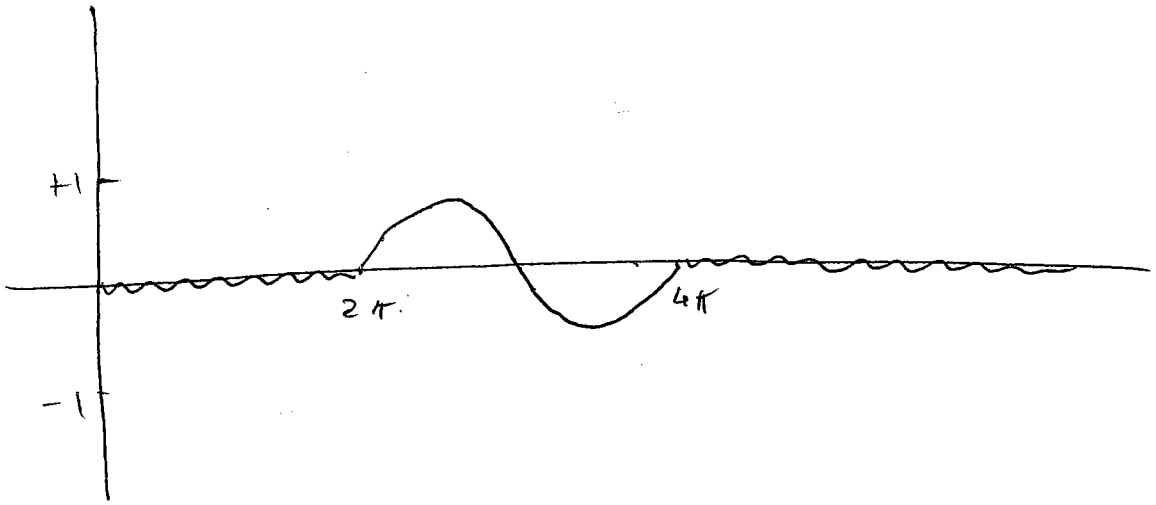
$$\Rightarrow \boxed{B=D=0, \quad A=-1, \quad C=1}$$

$$X(s) = \frac{-s}{s^2+5} + \frac{s}{s^2+4}$$

$$\Rightarrow \boxed{x(t) = \cos(2t) - \cos(\sqrt{5}t)}$$

iv) (20 Points) Let $f(t) = u(t-2\pi)\sin(t-2\pi) - u(t-4\pi)\sin(t-4\pi)$.

a) Sketch the graph of $f(t)$.



b) Find the Laplace Transform of $f(t)$.

$$\mathcal{L}\{f(t)\} = e^{-2\pi s}\cdot\frac{1}{s^2+1} - e^{-4\pi s}\frac{1}{s^2+1}$$

$$= \frac{e^{-2\pi s} - e^{-4\pi s}}{s^2+1}$$

v) (15 Points) State whether the following statements are true or false. Show your answer by making a circle on TRUE or FALSE.

a) $\mathcal{L}\{t\sin(t)\} = \frac{2s}{(s^2+1)^2}$ (TRUE) FALSE

b) $\mathcal{L}^{-1}\{\ln(s+3)\} = -\frac{1}{t}e^{-3t}$ (TRUE) FALSE

c) Let $f(t) = 1$ and $g(t) = 1$ then the convolution product is

$$(f * g)(t) = t \quad \text{(TRUE)} \quad \text{FALSE}$$

d) If $f(t)$ is a periodic function with period p and $g(t)$ is another function such that $g'(t) = f(t)$ then $g(t)$ is also periodic with period p. TRUE (FALSE)

e) $\mathcal{L}\{f(t).g(t)\} = \mathcal{L}\{f(t)\}.g(t) + f(t).\mathcal{L}\{g(t)\}$ TRUE (FALSE)

vi) (5 Points) Find $\mathcal{L}\{te^t\sin(5t)\}$.

$$= -\frac{d}{ds}\left[\mathcal{L}\{e^t\sin(5t)\}\right] = -\left[\frac{5}{(s-1)^2+25}\right]'$$

$$= -5\left[\frac{-2(s-1)}{((s-1)^2+25)^2}\right] = \frac{10(s-1)}{[(s-1)^2+25]^2}$$