Math 3113, Quiz I

January 31, 2007

1. (7 points)

   (a) Is $x^2y' = 1 - x^2 + y^2 - x^2y^2$ a separable differential equation ? If so, then separate it [write it in the form (terms involving y) $\times$ y' =(terms involving x)]. Do not solve the differential equation.

   Factorize the right hand side of the differential equation to get $x^2y' = (1-x^2)(1+y^2)$ and upon rearranging the terms we get

$$\frac{1}{1+y^2}y' = \frac{1-x^2}{x^2}$$

   Hence we conclude that the differential equation is separable.

   (b) Is $xy' + (2x-3)y = 4x^4$ a separable differential equation ? If so, then separate it [write it in the form (terms involving y) $\times$ y' =(terms involving x)]. Do not solve the differential equation.

   No amount of manipulation can lead to separation of variables. Hence the differential equation is not separable.

2. (7 points)

   (a) Is $x^2y' = 1 - x^2 + y^2 - x^2y^2$ a linear first order differential equation ? If so, write it in the standard for $y' + P(x)y = Q(x)$ and find the integrating factor. Do not solve the differential equation.

   The y term occurs with an exponent 2. In a linear first order differential equation we are not allowed that. Hence the differential equation is not linear first order.

(b) Is $xy' + (2x-3)y = 4x^4$ a linear first order differential equation ? If so, write it in the standard form $y' + P(x)y = Q(x)$ and find the integrating factor. Do not solve the differential equation.

The differential equation only has the first derivative and the y terms do not occur with higher powers. Hence this is a linear first order differential equation. To get it in the standard form divide both sides by x to get $y' + \left(\frac{2x-3}{x}\right)y = 4x^3$. To get the integrating factor

$$\rho(x) = e^{\int\left(\frac{2x-3}{x}\right)dx} = e^{\int\left(2-\frac{3}{x}\right)dx} = e^{2x-3\ln x} = e^{2x}x^{-3}$$

3. (6 points) Is there a value of the constant C for which $y(x) = C(e^x + x)$ is a solution of the differential equation $y' + y = 0$? Explain your answer.

If one sets $C = 0$ then we immediately get $y(x) = 0$ and hence $y'(x) = 0$ which certainly satisfies the relation $y' + y = 0$. So, just by inspection we can see that $C = 0$ gives a solution to the differential equation. A systematic analysis can show that $C = 0$ is the only value for which $y(x)$ is a solution :

$y'(x) = C(e^x + 1) \implies y' + y = C(e^x + 1) + C(e^x + x) = C(2e^x + x + 1)$. Now $y' + y = 0$ implies that

$$C(2e^x + x + 1) = 0 \text{ for all values of } x$$

which is possible if and only if $C = 0$.