Math 3113, Quiz II

February 14, 2007

1. (5 points) : Rewrite the homogeneous differential equation

$$yx^2y' = y^2x + \sqrt{x^3y^3}$$

in the standard form $y' = F(y/x)$. Do not solve.

Divide both sides by yx^2 to get

$$y' = \frac{y^2x}{yx^2} + \frac{1}{yx^2}\sqrt{x^3y^3} = \frac{y}{x} + \sqrt{\frac{x^3y^3}{y^2x^4}} = \frac{y}{x} + \sqrt{\frac{y}{x}}$$

as required.

2. (5 points) : Show that

$$(\cos x + \ln y)dx + (\frac{x}{y} + e^y)dy = 0$$

is an exact differential equation. Do not solve.

If the differential equation is given by $Mdx + Ndy = 0$ then it is an exact differential equation if it satisfies $M_y = N_x$. In this case $M = \cos x + \ln y$ and $N = \frac{x}{y} + e^y$. Hence

$$M_y = \frac{1}{y}, \qquad N_x = \frac{1}{y} \implies M_y = N_x$$

as required.

3. (5 points) : Are the two functions $y_1(x) = (\cos x)^2, y_2(x) = 1+\cos 2x$ linearly dependent or linearly independent ? Explain your answer.

If we notice the trignometric identity $(\cos x)^2 = \frac{1}{2}(1+\cos 2x)$ then we can immediately write

$$(1)y_1 + (-\frac{1}{2})y_2 = 0$$

which implies that the two functions are linearly dependent. If we start with evaluating the Wronskian of the two functions we get that $W(y_1, y_2)$ is equal to

$$-2(\cos x)^2 \sin 2x + 2\sin x \cos x(1+\cos 2x) = \sin 2x(-2(\cos x)^2 + (1+\cos 2x)) = 0$$

We have used the two identities $\sin 2x = 2\sin x \cos x$ and $(\cos x)^2 = \frac{1}{2}(1+\cos 2x)$. Since the Wronskian is identically zero, the two functions are linearly dependent.

4. (5 points) : Find the general solution to the differential equation

$$y^{(3)} - 4y'' + 4y' = 0.$$

The characteristic equation is given by

$$\begin{aligned} r^3 - 4r^2 + 4r &= 0 \\ \Longrightarrow r(r-2)^2 &= 0 \end{aligned}$$

Hence the roots of the characteristic equation are $r = 0$ once and $r = 2$ repeated twice. Hence the general solution is

$$y(x) = C_1 + (C_2 + C_3 x)e^{2x}.$$