Solution to Home work 4

Sec 3.1

(24) $f(x) = \sin^2 x$, $g(x) = 1 - \cos(2x)$

Double angle formula $\cos(2x) = 1 - 2\sin^2 x$

$\Rightarrow$ $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$

$\Rightarrow$ $(1)\sin^2(x) + \left(\frac{-1}{2}\right)(1 - \cos(2x)) = 0$ $\Rightarrow$ $\boxed{\text{lin. dependent.}}$

(26) $f(x) = 2\cos x + 3\sin x$, $g(x) = 3\cos x - 2\sin x$

If $f(x) = A g(x)$ $\Rightarrow$ $2\cos x + 3\sin x = A(3\cos x - 2\sin x)$

$\Rightarrow$ $2 = 3A$, $3 = -2A$ $\Rightarrow$ $A = \frac{2}{3}$ & $A = \frac{-3}{2}$ not possible

$\Rightarrow$ f, g are $\boxed{\text{linearly independent}}$

(34) $y'' + 2y' - 15y = 0$ Char eq^n $r^2 + 2r - 15 = 0 \Rightarrow (r+5)(r-3) = 0$

$\Rightarrow r = -5,\ r = 3$, $\Rightarrow$ general sol^n $\boxed{y(x) = c_1 e^{-5x} + c_2 e^{3x}}$

(40) $9y'' - 12y' + 4y = 0$ Char eq^n $9r^2 - 12r + 4 = 0 \Rightarrow (3r-2)^2 = 0$

$\Rightarrow$ $r = 2/3$ repeated twice $\Rightarrow$ general sol^n $\boxed{y(x) = (c_1 + c_2 x)e^{2/3 x}}$

(46) $y(x) = c_1 e^{10x} + c_2 e^{100x}$ $\Rightarrow$ roots of char eq^n : $r = 10$, $r = 100$

$\Rightarrow$ char eq^n is $(r-10)(r-100) = r^2 - 110r + 1000 = 0$

$\Rightarrow$ D.E is $\boxed{y'' - 110y' + 1000y = 0}$

Sec 3.2

(1) $f(x) = 2x$, $g(x) = 3x^2$, $h(x) = 5x - 8x^2$

$$\left(\frac{5}{2}\right)(2x) + \left(\frac{8}{3}\right)(3x^2) + (-1)(5x - 8x^2) = 0$$

(9) $f(x) = e^x$, $g(x) = e^{2x}$, $h(x) = e^{3x}$

$$W(f,g,h) = \begin{vmatrix} e^x & e^{2x} & e^{3x} \\ e^x & 2e^{2x} & 3e^{3x} \\ e^x & 4e^{2x} & 9e^{3x} \end{vmatrix} = e^x \begin{vmatrix} 2e^{2x} & 3e^{3x} \\ 4e^{2x} & 9e^{3x} \end{vmatrix} - e^{2x} \begin{vmatrix} e^x & 3e^{3x} \\ e^x & 9e^{3x} \end{vmatrix}$$

$$+ e^{3x} \begin{vmatrix} e^x & 2e^{2x} \\ e^x & 4e^{2x} \end{vmatrix}$$

$$= e^x(\cancel{18}\, 6e^{5x}) - e^{2x}(6e^{4x})$$

$$+ e^{3x}(2e^{3x}) \quad = 2e^{6x} \neq 0 \quad \text{for all } x$$

(14) $y^{(3)} - 6y'' + 11y' - 6y = 0$; $y(0) = 0$, $y'(0) = 0$, $y''(0) = 3$

$y(x) = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} \Rightarrow y'(x) = c_1 e^x + 2c_2 e^{2x} + 3c_3 e^{3x}$

$y''(x) = c_1 e^x + 4c_2 e^{2x} + 9c_3 e^{3x}$

$\Rightarrow 0 = y(0) = c_1 + c_2 + c_3$ — (1)

$0 = y'(0) = c_1 + 2c_2 + 3c_3$ — (2)

$3 = y''(0) = c_1 + 4c_2 + 9c_3$ — (3)

(2) − (1) : $0 = c_2 + 2c_3$

(3) − (2) : $3 = 2c_2 + 6c_3$

$c_2 = -3$, $c_3 = \frac{3}{2}$, $c_1 = \frac{3}{2}$

$$\Rightarrow \boxed{y(x) = \frac{3}{2}e^x - 3e^{2x} + \frac{3}{2}e^{3x}}$$

(18) $y^{(3)} - 3y'' + 4y' - 2y = 0$; $y(0) = 1$, $y'(0) = 0$, $y''(0) = 0$

$$y(x) = c_1 e^x + c_2 e^x \sin x + c_3 e^x \cos x$$

the initial conditions gives us $c_1 + c_2 = 1$, $c_1 + c_2 + c_3 = 0$,

$c_1 + 2c_3 = 0 \Rightarrow c_1 = 2,\ c_2 = -1,\ c_3 = -1$ Hence

$$\boxed{y(x) = 2e^x - e^x \sin x - e^x \cos x}.$$

(23) $y'' - 2y' - 3y = 6 \qquad y(0) = 3,\ y'(0) = 11$

$y(x) = y_c + y_p = c_1 e^{-x} + c_2 e^{3x} - 2$

$\Rightarrow y'(x) = -c_1 e^{-x} + 3c_2 e^{3x}$. $\qquad 3 = y(0) = c_1 + c_2 - 2$

$11 = y'(0) = -c_1 + 3c_2$

$\Rightarrow c_1 = 1,\ c_2 = 4$

$$\Rightarrow \boxed{y(x) = e^{-x} + 4e^{3x} - 2}$$

Sec 3.3

(10) $5y^{(4)} + 3y^{(3)} = 0 \qquad$ Char eq^n: $5r^4 + 3r^3 = 0$

$\Rightarrow r^3(5r+3) = 0 \Rightarrow$ roots $r = 0, 0, 0, -3/5$

General sol^n $\boxed{y(x) = c_1 e^{-3/5 x} + c_2 + c_3 x + c_4 x^2}$

(11) $y^{(4)} - 8y^{(3)} + 16y'' = 0 \qquad$ Char eq^n: $r^4 - 8r^3 + 16r^2 = 0$

$\Rightarrow r^2(r^2 - 8r + 16) = 0 \Rightarrow r^2(r-4)^2 = 0 \Rightarrow$ roots $r = 0, 0, 4, 4$

General sol^n $\boxed{y(x) = (c_1 + c_2 x) + (c_3 + c_4 x)e^{4x}}$

(21) $y'' - 4y' + 3y = 0$; $y(0) = 7$, $y'(0) = 11$

Char eq^n: $r^2 - 4r + 3 = 0 \Rightarrow (r-3)(r-1) = 0 \Rightarrow r = 3, 1$

General sol^n $y(x) = C_1 e^x + C_2 e^{3x} \Rightarrow y_1'(x) = C_1 e^x + 3C_2 e^{3x}$

$\Rightarrow 7 = y(0) = C_1 + C_2$, $11 = y'(0) = C_1 + 3C_2 \Rightarrow \boxed{C_2 = 2,\ C_1 = 5}$

$\Rightarrow \boxed{y(x) = 5e^x + 2e^{3x}}$

(25) $3y^{(3)} + 2y'' = 0$; $y(0) = -1$, $y'(0) = 0$, $y''(0) = 1$

Char eq^n: $3r^3 - 2r^2 = 0 \Rightarrow r^2(3r-2) = 0 \Rightarrow r = 0, 0, 2/3$

$\Rightarrow$ General sol^n : $y(x) = C_1 + C_2 x + C_3 e^{2/3 x}$

$\Rightarrow y'(x) = C_2 + \frac{2}{3} C_3 e^{2/3 x}$, $y_2'' = \frac{4}{9} C_3 e^{4/3 x}$

$-1 = y(0) = C_1 + C_3$, $0 = y'(0) = C_2 + \frac{2}{3} C_3$, $1 = y''(0) = \frac{4}{9} C_3$

$\Rightarrow C_3 = \frac{9}{4}$, $C_2 = -\frac{2}{3}$, $C_1 = -\frac{1}{3}$

$\Rightarrow \boxed{y(x) = -\frac{1}{3} - \frac{2}{3}x + \frac{9}{4} e^{2/3 x}}$

(28) $2y^{(3)} - y'' - 5y' - 2y = 0$ Char eq^n $2r^3 - r^2 - 5r - 2 = 0$

$r = 2$ is a root by inspection : $(r-2)(2r^2 + 3r + 1) = 0$

$(r+2)(r+1)(2r+1) = 0 \Rightarrow r = 2, r = -1, r = -1/2$

General sol^n : $\boxed{y(x) = C_1 e^{2x} + C_2 e^{-x} + C_3 e^{1/2 x}}$