Solution to Homework 5

Section 3.3 :

(5) $y'' + 6y' + 9y = 0$: $r^2 + 6r + 9 = 0 \Rightarrow (r+3)^2 = 0 \Rightarrow r = -3, -3$

$\Rightarrow \boxed{y(x) = (C_1 + C_2 x)e^{-3x}}$

(18) $y^{(4)} = 16y$: $r^4 = 16 \Rightarrow r^4 - 16 = 0 \Rightarrow (r^2-4)(r^2+4) = 0$

$\Rightarrow (r+2)(r-2)(r^2+4) = 0 \Rightarrow r = \pm 2, \pm 2i$

$\Rightarrow \boxed{y(x) = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos(2x) + C_4 \sin(2x)}$

(26) $y^{(3)} + 10y'' + 25y' = 0$; $y(0) = 3$, $y'(0) = 4$, $y''(0) = 5$

$r^3 + 10r^2 + 25r = 0 \Rightarrow r(r+5)^2 = 0 \Rightarrow r = 0, -5, -5$

$\Rightarrow y(x) = C_1 + (C_2 + C_3 x)e^{-5x}$, $y'(x) = C_3 e^{-5x} - 5(C_2 + C_3 x)e^{-5x}$

$y''(x) = -10 C_3 e^{-5} + 25(C_2 + C_3 x)e^{-5x}$

$3 = y(0) = C_1 + C_2$, $4 = y'(0) = C_3 - 5C_2$, $5 = y''(0) = -10C_3 + 25C_2$

$\Rightarrow C_1 = \frac{24}{5}$, $C_2 = -\frac{9}{5}$, $C_3 = -5$ $\Rightarrow \boxed{y(x) = \frac{24}{5} + \left(-\frac{9}{5} - 5x\right)e^{-5x}}$

(40) $y(x) = Ae^{2x} + B\cos(2x) + C\sin(2x)$: $r = 2, \pm 2i$

$\Rightarrow$ Char eq^n : $(r-2)(r^2+4) = r^3 - 2r^2 + 4r - 8 = 0$

$\Rightarrow \boxed{y^{(3)} - 2y'' + 4y' - 8y = 0}$

(42) $y(x) = (A + Bx + Cx^2)\cos(2x) + (D + Ex + Fx^2)\sin(2x)$: $r = \pm 2i, \pm 2i, \pm 2i$

$\Rightarrow$ char eq^n : $(r^2+4)^3 = r^6 + 12r^4 + 48r^2 + 64 = 0$

$\Rightarrow \boxed{y^{(6)} + 12y^{(4)} + 48y'' + 64y = 0}$

Section 3.5 :

(6) $2y'' + 4y' + 7y = x^2$: $y_p = A + Bx + Cx^2$

$\Rightarrow 7A + 4B + 4C = 0,\ 7B + 8C = 0,\ 7C = 1 \Rightarrow \boxed{y_p = \frac{4}{343} - \frac{8}{49}x + \frac{1}{7}x^2}$

(17) $y'' + y = \sin x + x\cos x$

We have $y_c = c_1\cos x + c_2\sin x$

First guess $y_p = (A + Bx)\cos x + (C + Dx)\sin x$

duplication $\Rightarrow$ Take $y_p = x(A + Bx)\cos x + x(C + Dx)\sin x$

$\Rightarrow 2B + 2C = 0,\ 4D = 1,\ -2A + 2D = 1,\ -4B = 0$

$\Rightarrow \boxed{y_p = \frac{x^2}{4}\sin x - \frac{x}{4}\cos x}$

(20) $y^{(3)} - y = e^x + 7$ We have $y_c = (c_1 + c_2x + c_3x^2)e^x$

First guess $y_p = Ae^x + B$ duplication $\Rightarrow$ Take $y_p = xAe^x + B$

$\Rightarrow -B = 7,\ 3A = 1 \Rightarrow \boxed{y_p = \frac{1}{3}xe^x - 7}$

(24) $y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$

y_c: $r^3 - r^2 - 12r = 0 \Rightarrow r(r-4)(r+3) = 0 \Rightarrow y_c = c_1 + c_2e^{4x} + c_3e^{-3x}$

First guess $y_p = (A + Bx) + (C + Dx)e^{-3x}$

Duplication $\Rightarrow \boxed{y_p = x(A + Bx) + x(C + Dx)e^{-3x}}$

(28) $y^{(4)} + 9y'' = (x^2 + 1)\sin(3x)$ y_c: $r^4 + 9r^2 = 0 \Rightarrow r^2(r^2 + 9) = 0$

$\Rightarrow y_c = (c_1 + c_2x) + (c_3\cos(3x) + c_4\sin(3x))$ First guess :

$y_p = (A + Bx + Cx^2)\cos(3x) + (D + Ex + Fx^2)\sin(3x)$

Duplication $\Rightarrow$ $\boxed{y_p = x(A+Bx+Cx^2)\cos(3x) + x(D+Fx+Ex^2)\sin(3x)}$

(34) $y'' + y = \cos x$; $y(0) = 1$, $y'(0) = -1$

y_c : $r^2 + 1 = 0$ $\Rightarrow$ $y_c(x) = c_1 \cos x + c_2 \sin x$

First guess : $y_p = A\cos x + B\sin x$ duplication $\Rightarrow$ Take $y_p = Ax\cos x + Bx\sin x$

Substitute y_p in DE to get $y_p = \frac{1}{2} x \sin x$

$\Rightarrow$ General sol^n $y(x) = y_c + y_p = c_1 \cos x + c_2 \sin x + \frac{1}{2} x \sin x$

$y(0) = 1$, $y'(0) = -1$ $\Rightarrow$ $c_1 = 1$, $c_2 = -1$

$\Rightarrow$ $\boxed{y(x) = \cos x - \sin x + \frac{1}{2} x \sin x}$

(39) $y^{(3)} + y'' = x + e^{-x}$; $y(0) = 1$, $y'(0) = 0$, $y''(0) = 1$

y_c : $r^3 + r^2 = 0$ $\Rightarrow$ $r^2(r+1) = 0$ $\Rightarrow$ $r = 0, 0, -1$

$\Rightarrow$ $y_c = c_1 + c_2 x + c_3 e^{-x}$

First guess y_p : $(A + Bx) + Ce^{-x}$ Duplication $\Rightarrow$ $y_p = x^2(A + Bx) + xCe^{-x}$

Substitute y_p in D.E. to get $y_p = -\frac{x^2}{2} + \frac{x^3}{6} + xe^{-x}$

$\Rightarrow$ General sol^n $y(x) = c_1 + c_2 x + c_3 e^{-x} - \frac{x^2}{2} + \frac{x^3}{6} + xe^{-x}$

Initial conditions $\Rightarrow$ $c_1 + c_2 = 1$, $c_2 - c_3 = 0$, $c_3 - 3 = 1$

$\Rightarrow$ $\boxed{y(x) = -3 + 3x - \frac{x^2}{2} + \frac{x^3}{6} + 4e^{-x} + xe^{-x}}$

(40) $y^{(4)} - y = 5$; $y(0) = y'(0) = y''(0) = y'''(0) = 0$

y_c: $r^4 - 1 = 0 \Rightarrow r = \pm 1, \pm i \Rightarrow y_c = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$

$y_p = A$ no duplication; Substitute y_p in D.E. to

get $y_p = -5 \Rightarrow$ General sol^n $y(x) = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x - 5$

Initial conditions $\Rightarrow$

$c_1 + c_2 + c_3 - 5 = 0$, $-c_2 + c_1 + c_4 = 0$,

$c_1 + c_2 - c_3 = 0$, $-c_2 + c_1 - c_4 = 0$ $\Rightarrow c_1 = c_2 = \frac{5}{4}$, $c_3 = \frac{5}{2}$, $c_4 = 0$

$$\Rightarrow \boxed{y(x) = \frac{5}{4} e^x + \frac{5}{4} e^{-x} + \frac{5}{2} \cos x - 5}$$