Homework 10 **:** This homework is due on **November** 15.

1. Which of the following functions are linear transformations ?
   (a) $L : R_2 \to R_3$ defined by $L(\begin{bmatrix} u_1 & u_2 \end{bmatrix}) = \begin{bmatrix} u_1 + 3u_2 & -u_2 & u_1 - u_2 \end{bmatrix}$.
   (b) $L : R_3 \to R_3$ defined by $L(\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}) = \begin{bmatrix} 2u_1 & u_1^2 + u_2^2 & 2u_3^2 \end{bmatrix}$.
   (c) $L : R_3 \to R_3$ defined by $L(\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}) = \begin{bmatrix} u_1 & 0 & u_3 \end{bmatrix}$.
   (d) $L : P_2 \to P_3$ defined by $L(p(t)) = t^3 p'(0) + t^2 p(0)$. (Here $p'(t)$ stands for derivative)
   (e) $L : M_{nn} \to M_{nn}$ defined by $L(A) = A^{-1}$.
   (f) Fix a 3×3 matrix X. Let $L : M_{33} \to M_{33}$ be defined by $L(A) = AX - XA$.

2. Find the standard matrix representing the linear transformation L in each of the following problems
   (a) $L : R^3 \to R_2$ defined by $L(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}) = \begin{bmatrix} u_2 + 5u_3 \\ 2u_1 \end{bmatrix}$.
   (b) $L : R^2 \to R_2$ defined by $L(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}) = \begin{bmatrix} u_1 + ku_2 \\ u_2 \end{bmatrix}$.
   (c) $L : R^3 \to R^3$ defined by $L(\mathbf{u}) = k\mathbf{u}$.

3. Suppose the standard matrix representing $L : R^3 \to R^3$ is given by $\begin{bmatrix} 0 & -2 & 3 \\ 1 & -4 & 2 \\ -2 & 0 & 1 \end{bmatrix}$. Find $L(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix})$ and $L(\begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix})$.

4. Let $L : P_2 \to R^1$ be the linear transformation given by $L(t^2+2) = 5, L(2t-1) = 1$ and $L(t^2+t+1) = -2$. Find the value of $L(2t^2 + 5t + 4)$ and $L(at^2 + bt + c)$.

5. Let $L : R^2 \to R^2$ be the linear transformation given by $L(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$. Which one of $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is in $Ker(L)$? Find a basis for and dimension of $Ker(L)$.

6. Let $L : R_4 \to R_2$ be the linear transformation defined by $L(\begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}) = \begin{bmatrix} u_1 - u_4 & u_2 + u_3 \end{bmatrix}$. Find a basis for and dimension of $Ker(L)$. Is L one-to-one ?

7. Let $L : R_2 \to R_3$ be a linear transformation defined by $L(\begin{bmatrix} u_1 & u_2 \end{bmatrix}) = \begin{bmatrix} u_2 & u_1 + u_2 & u_1 \end{bmatrix})$. Find $Ker(L)$. Is L one-to-one ?

8. Let $L : M_{22} \to M_{22}$ be a linear transformation defined by $L(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = \begin{bmatrix} a+b & b+c \\ a+d & b+d \end{bmatrix}$. Find a basis for and dimension of $Ker(L)$.

9. Find a basis for and dimension of the kernel of $L : R^5 \to R^4$ where

$$L(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}) = \begin{bmatrix} 1 & 0 & -1 & 3 & 1 \\ 1 & 0 & 0 & 2 & -1 \\ 2 & 0 & -1 & 5 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$$