Homework 2 : This homework is due on **September** 6.

1. Find the inverse (if possible) of

   (a) $A = \begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$

   (b) $(AB^T)^{-1}$ with A, B as above.

2. The linear system $AC\mathbf{x} = \mathbf{b}$ is such that A and C are nonsingular with $A^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$, $C^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Find the solution $\mathbf{x}$.

3. Consider the linear system $A\mathbf{x} = \mathbf{b}$, where A is the matrix from Problem $1(a)$.

   (a) Find a solution if $\mathbf{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

   (b) Find a solution if $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

4. Let A and B be symmetric matrices.

   (a) Show that $A + B$ is symmetric.

   (b) Show that AB is symmetric if and only if $AB = BA$.

5. If A is a skew-symmetric matrix, what type of matrix is A^T ? Justify your answer.

6. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be the matrix transformation defined by $f(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$. Determine which of the given vectors is in the range of f.

$$\mathbf{w_1} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{w_2} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{w_3} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

7. Let $f(\mathbf{x}) = A\mathbf{x}$ be a matrix transformation with $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \end{pmatrix}$. Find two different vectors $\mathbf{u}$ and $\mathbf{v}$ such that $f(\mathbf{u}) = \mathbf{w}$ and $f(\mathbf{v}) = \mathbf{w}$ where $\mathbf{w} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

8. Give a geometric description of the matrix transformation $f(\mathbf{u}) = A\mathbf{u}$ for the following matrices A.

$$A_1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

9. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the matrix transformation defined by $f(\mathbf{u}) = A\mathbf{u}$ with $A = \begin{pmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{pmatrix}$. For $\phi = 30$ degrees, f defines a clockwise rotation by an angle of 30 degrees.

   (a) If $T_1(\mathbf{u}) = A^2\mathbf{u}$ describe the action of T_1 on $\mathbf{u}$.

   (b) If $T_2(\mathbf{u}) = A^{-1}\mathbf{u}$ describe the action of T_1 on $\mathbf{u}$.

   (c) What is the smallest positive integer k for which $T(\mathbf{u}) = A^k\mathbf{u} = \mathbf{u}$?