Homework 5 **:** This homework is due on **October** 4.

1. State whether the following statement is true or false and explain your answer : If $\det(AB) = 0$ then either $\det(A) = 0$ or $\det(B) = 0$.

2. Suppose A is a $n \times n$ matrix and k is a non-zero real number. Show that $\det(kA) = k^n \det(A)$.

3. Suppose A is a $n \times n$ skew-symmetric matrix with n odd. Show that $\det(A) = 0$.

4. Let $A = \begin{pmatrix} 1 & 0 & 3 & 0 \\ 2 & 1 & -4 & -1 \\ 3 & 2 & 4 & 0 \\ 0 & 3 & -1 & 0 \end{pmatrix}$. Find the cofactors $A_{12}, A_{23}, A_{33}, A_{41}$.

5. Find all the values of t for which
$$\det(\begin{pmatrix} t-1 & 0 & 1 \\ -2 & t+2 & -1 \\ 0 & 0 & t+1 \end{pmatrix}) = 0.$$

6. Let $A = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 0 \\ 3 & -2 & 1 \end{pmatrix}$. Find $adjA$. Compute $\det(A)$. Verify that $A(adjA) = \det(A)I_3$.

7. Let $A = \begin{pmatrix} 2 & -1 & 0 & 3 \\ 1 & 2 & -2 & 4 \\ -1 & 1 & -3 & -2 \\ 0 & 2 & -1 & 5 \end{pmatrix}$. Find $\det(A)$ by expanding along a row or column. (You might want to simplify A first by using elementary row or column operations)

8. Using the method of adjoint matrix find the inverse of $A = \begin{pmatrix} 4 & 2 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{pmatrix}$.

9. Suppose A is a $n \times n$ non-singular matrix. Show that $\det(adjA) = \det(A)^{n-1}$.