This homework is due on **October** 11.

1. Show that if $A^n = 0$ for some positive integer n (such a matrix A is called a nilpotent matrix) then $\det(A) = 0$.

2. Use Cramer's rule to solve the following linear system

$$2x_1 + 4x_2 + 6x_3 = 2, \quad x_1 + 2x_3 = 0, \quad 2x_1 + 3x_2 - x_3 = -5$$

3. Use Cramer's rule to solve the following linear system

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 1 & 0 & -2 & 4 \\ -1 & 5 & 2 & 0 \\ 0 & 2 & -1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 4 \\ 0 \end{pmatrix}$$

4. Suppose V is the set of all 2×2 matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $abcd = 0$. Let the operation $\oplus$ be the standard addition of matrices and the operation $\odot$ be the standard scalar multiplication of matrices.

   (a) Is V closed under addition ?
   (b) Is V closed under scalar multiplication ?
   (c) What is the zero vector in the set V ?
   (d) Does every matrix A in V have a negative that is in V ? Explain.
   (e) Is V a vector space ? Explain.

5. Let V be the set of 2×2 matrices $A = \begin{pmatrix} a & b \\ 2b & d \end{pmatrix}$. Let the operation $\oplus$ be the standard addition of matrices and the operation $\odot$ be the standard scalar multiplication of matrices.

   (a) Is V closed under addition ?
   (b) Is V closed under scalar multiplication ?
   (c) What is the zero vector in the set V ?
   (d) Does every matrix A in V have a negative that is in V ? Explain.
   (e) Is V a vector space ? Explain.

6. Let V be the set of all 2×1 matrices $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $x \leq 0$ with the usual operations in R^2. Is V a vector space ? If not, state which of the properties in the definition of a vector space do not hold.

7. Let V be the set of real numbers; define $\mathbf{u} \oplus \mathbf{v} = \mathbf{uv}$ (ordinary multiplication) and $c \odot \mathbf{u} = c + \mathbf{u}$. Is V a vector space ? If not, state which of the properties in the definition of a vector space do not hold.

8. Let V be the set of all positive real numbers; define $\mathbf{u} \oplus \mathbf{v} = \mathbf{uv}$ (ordinary multiplication) and $c \odot \mathbf{u} = \mathbf{u}^c$. Is V a vector space ? If not, state which of the properties in the definition of a vector space do not hold.