Homework 7 : This homework is due on **October** 18.

1. In the following problems you are given a vector space V and a subset W. You have to answer whether W is a vector subspace of V. Explain your answer.

   (a) $V = R^3$ and W is set of all vectors of the form $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ with $2a - b + 5c = 0$.

   (b) $V = R^3$ and W is set of all vectors of the form $\begin{pmatrix} a \\ 2 \\ c \end{pmatrix}$

   (c) $V = M_{23}$ and W is the set of all matrices of the form $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ such that $b = 2a$ and $e = f - 3d$.

   (d) $V = M_{22}$ and W is the set of all matrices A such that $A \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 0$.

   (e) $V = R^n$ Let A be a $n \times n$ matrix. Let W be the set of vectors $\mathbf{x}$ such that $A\mathbf{x} \neq 0$.

   (f) $V = M_{nn}$ and W is the set of all upper triangular matrices.

   (g) $V = M_{nn}$ and W is the set of all non-singular matrices.

   (h) $V = C(-\infty, \infty)$ the vector space of all functions on $(-\infty, \infty)$ and W is the set of functions with $f(0) = 0$.

   (i) $V = C(-\infty, \infty)$ the vector space of all functions on $(-\infty, \infty)$ and W is the set of functions with $f(0) = 5$.

2. Does the set $S = \{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\}$ span M_{22} ?

3. Find the set of vectors spanning the solution space of $A\mathbf{x} = 0$, where $A = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 2 & 3 & 6 & -2 \\ -2 & 1 & 2 & 2 \\ 0 & -2 & -4 & 0 \end{pmatrix}$.

4. Determine whether $p(t) = 2t^2 + 2t + 3$ belongs to the span of $p_1(t) = t^2 + 2t + 1, p_2(t) = t^2 + 3$ and $p_3(t) = t - 1$.

5. Let $A_1 = \begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix}, A_2 = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$, and $A_3 = \begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}$. Determine whether $A = \begin{pmatrix} 5 & 1 \\ -1 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ belong to the span of A_1, A_2, A_3.