Homework 8 **:** This homework is due on **November** 1.

i) State whether the following vectors are linearly independent. If not, express one vector as a linear combination of the rest.

a) $V = R_3, \quad \mathbf{v}_1 = (1,1,0), \mathbf{v}_2 = (0,3,4), \mathbf{v}_3 = (2,0,4), \mathbf{v}_4 = (1,1,1)$

b) $V = R_3, \quad \mathbf{v}_1 = (2,-1,3), \mathbf{v}_2 = (4,1,2)$

c) $V = M_{22}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 4 & 6 \\ 8 & 6 \end{bmatrix}$

d) $V = M_{22}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

e) $V = P_2, \quad \mathbf{v}_1 = 2t^2 + t + 1, \mathbf{v}_2 = 3t^2 + t - 5, \mathbf{v}_3 = t + 13$

ii) For what values of c are the vectors $(-1,0,-1), (2,1,2), (1,1,c)$ in R_3 linearly dependant ?

iii) Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors in a vector space V. Prove that $T = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ is also linearly independent, where $\mathbf{w}_1 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3, \mathbf{w}_2 = \mathbf{v}_2 + \mathbf{v}_3$ and $\mathbf{w}_3 = \mathbf{v}_3$.

iv) Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors in a vector space V. Is $T = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$, where $\mathbf{w}_1 = \mathbf{v}_1 + \mathbf{v}_2, \mathbf{w}_2 = \mathbf{v}_2 - \mathbf{v}_3$ and $\mathbf{w}_3 = \mathbf{v}_1 + \mathbf{v}_3$, linearly dependent or linearly independent. Justify your answer.

v) Which of the following set of vectors are bases ?

a) $V = R^3 \quad \{\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\}$

b) $V = P_3 \quad \{t^3 + t^2 + 1, t^3 - 1, t^3 + t^2 + t\}$

c) $V = R_4 \quad \{(-2,4,6,4), (0,1,2,0), (-1,2,3,2), (-3,2,5,6), (-2,-1,0,4)\}$

vi) Find a basis for the subspace W of R_4 spanned by the set of vectors $\{(1,1,0,-1), (0,1,2,1), (1,0,1,-1), (1,1,-6,-3), (-1,-5,1,0)\}$.

vii) Let W be the subspace of P_3 spanned by $\{t^3+t^2-2t+1, t^2+1, t^3-2t, 2t^3+3t^2-4t+3\}$. Find a basis for W.

viii) Find all values of a for which $\{(a^2,0,1), (0,a,2), (1,0,1)\}$ is a basis for R_3.