Math 3333, Quiz V, October 30, 2007

Name :

i) (15 Points) Let W be the subspace of M_{22} of all matrices of the form $\begin{bmatrix} a & 2a \\ c & d \end{bmatrix}$. Is $S = \{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\}$ a basis for W ? Explain your answer.

We have to check two things :

a) **Span :** $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{v}$ gives us

$$\begin{bmatrix} a_3 & 2a_3 \\ a_2 & a_1 \end{bmatrix} = \begin{bmatrix} a & 2a \\ c & d \end{bmatrix}.$$

Hence we get $a_1 = d, a_2 = c, a_3 = a$ and $Span(S) = W$.

b) **Linear Independence :** $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = 0$ gives us

$$\begin{bmatrix} a_3 & 2a_3 \\ a_2 & a_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Hence we get $a_1 = a_2 = a_3 = 0$ which implies that S is linearly independent.

This tells us that, indeed, S is a basis for W.

ii) (5 Points) Write down a basis for $V = M_{23}$. No proof required, just write down the basis vectors.

$$\{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\}$$