Math 3333, Quiz VII, November 27, 2007

Name :

Let $L : P_1 \to P_2$ be the linear transformation defined by $L(at+b) = (a+b)t^2 + (a-b)t + 3b$.

1. Find a basis for and the dimension of the range of L.
2. Let $S = \{t+2, t-1\}$ be a basis of P_1 and $T = \{t^2, t, 1\}$ be a basis of P_2. Find the matrix representation of L with respect to the ordered basis S and T.

Any vector $\mathbf{v}$ in the image of L has the form

$$\mathbf{v} = (a+b)t^2 + (a-b)t + 3b = a(t^2+t) + b(t^2 - t + 3)$$

Hence Range(L) is spanned by $\{t^2+t, t^2-t+3\}$. These two vectors are linearly independent, which implies that they form a basis for the Range of L. Hence, we get

$$\dim(\text{Range}(L)) = 2.$$

To find the matrix representation of L with respect to S and T, we first have to evaluate

$$L(t+2) = 3t^2 - t + 6 = 3(t^2) + (-1)(t) + 6(1) \Rightarrow [L(t+2)]_T = \begin{bmatrix} 3 \\ -1 \\ 6 \end{bmatrix}$$

$$L(t-1) = 2t - 3 = 0(t^2) + 2(t) + (-3)(1) \Rightarrow [L(t-1)]_T = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$$

Hence the matrix representing L with respect to S and T is

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 6 & -3 \end{bmatrix}.$$