MATH 3333 Quiz 1 Sep. 12, 2008

i) (10 Points) Find the solution $\mathbf{x}$ of the linear system

$$(AB^{-1}C^T)\mathbf{x} = \mathbf{b} \quad \text{where} \quad A^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 1 \\ 5 & 2 \end{bmatrix}, \quad C^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{aligned} (AB^{-1}C^T)\mathbf{x} = \mathbf{b} \Rightarrow \mathbf{x} &= (AB^{-1}C^T)^{-1}\mathbf{b} = (C^{-1})^T BA^{-1}\mathbf{b} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 25 & 10 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 7 & 5 \\ 60 & 35 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 12 \\ 95 \end{bmatrix} \end{aligned}$$

ii) (10 Points) Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 7 & -1 & 5 \\ 0 & 4 & 1 \end{bmatrix}$. Write A as $A = S + K$, where S is a symmetric matrix and K is a skew-symmetric matrix.

We have $S = \frac{1}{2}(A + A^T)$ and $K = \frac{1}{2}(A - A^T)$. This implies that

$$S = \begin{bmatrix} 2 & 4 & 3/2 \\ 4 & -1 & 9/2 \\ 3/2 & 9/2 & 1 \end{bmatrix} \qquad K = \begin{bmatrix} 0 & -3 & 3/2 \\ 3 & 0 & 1/2 \\ -3/2 & -1/2 & 0 \end{bmatrix}$$

iii) (5 Points) State which of the following matrices are in row echelon form.

$$A = \begin{bmatrix} 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A and C : row echelon matrix

B : not row echelon matrix