MATH 3333

Midterm I

October 3, 2008

Name :

I.D. no. :

- Calculators are not allowed. The problems are set so that you should not need calculators at all.

- Show as much work as possible. Answers without explanation will not receive any credit.

- If you preform any row or column operations in a problem, record them using standard notations.

- Best of Luck.

i) (20 Points) Find all the values of a for which the resulting linear system has (a) no solution, (b) a unique solution and (c) infinitely many solutions.

$$\begin{aligned} x + 2y - 3z &= 3 \\ x + 3y + z &= 1 \\ x + 2y + (a^2 - 7)z &= a + 1 \end{aligned}$$

ii) (20 Points) Find all 3×1 matrices $\mathbf{x}$ such that

$$A\mathbf{x} = 2\mathbf{x}, \qquad \text{where } A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 3 \end{bmatrix}.$$

iii) a) (5 Points) Determine whether the following permutations of $S = \{1, 2, 3, 4, 5, 6\}$ are even or odd. Explain why.

$(a)624513$ $\qquad\qquad$ $(b)536214$

b) (10 Points) Let A be a $n \times n$ skew-symmetric matrix. Show that for any even integer k, the matrix A^k is symmetric.

c) (5 Points)

$$\text{If } \det\left(\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\right), \text{ then find } \det\left(\begin{bmatrix} a_1 & a_2 & a_3 \\ 2b_1 & 2b_2 & 2b_3 \\ c_1 - a_1 & c_2 - a_2 & c_3 - a_3 \end{bmatrix}\right)$$

iv) (20 Points) Let

$$A = \begin{bmatrix} 2 & 3 & -1 & 0 \\ -3 & 0 & 5 & 1 \\ 1 & 1 & -4 & 2 \\ 0 & 2 & 4 & -1 \end{bmatrix}$$

a) Find Adj(A). (Use the fact that $A_{11} = 37, A_{12} = 17, A_{14} = -41, A_{21} = -21, A_{22} = -9, A_{24} = 21, A_{31} = 17, A_{32} = 7, A_{33} = -13, A_{34} = -19, A_{42} = 10, A_{43} = -22, A_{44} = -28$)

b) Find det(A)

c) Find A^{-1}, if it exists.

v) (20 Points) Find A^{-1}, if it exists, for

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$