Mathematics 2443-002 Name (please print) ______________________

Examination II Form A

October 23, 2000

I. (8) Consider the following minimization problem: A cardboard box without a lid is to have a volume of 6 cubic feet. Find the dimensions of the box that minimize the amount of cardboard used.

1. Sketch the box and label its dimensions as x, y, and z, in such a way that the dimensions of the base are y by z.

2. Using the method of Lagrange multipliers, write four equations in the four unknowns x, y, z, and λ, so that one of the solutions of the equations corresponds to the box of minimum cardboard, but *do not* try to solve the equations or proceed further with determining the point that minimizes.

II. (8) Sketch the region of integration, and change the order of integration for $\int_0^1 \int_{x^2}^{2-x} f(x, y)\, dy\, dx$.

III. (8) This problem concerns the polar equation $r = \sin(3\theta)$.

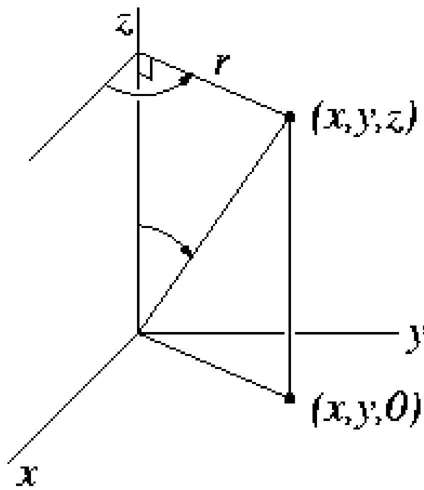
1. For $0 \le \theta \le 2\pi$, make two graphs of $r = \sin(3\theta)$, one in the (r, θ)-plane and one in the (x, y)-plane.

2. Write a double integral in polar coordinates whose value is the area of the part of the plane enclosed by the loops of this curve. Supply limits of integration, but *do not* go on to calculate the value of the integral.

IV. (6) Calculate $\iiint_E xyz\,dV$, where E is the region in the first quadrant lying above the square $[0,1] \times [0,1]$ in the xy-plane and below the plane $z = 2x$.

V. Recall that in spherical coordinates, $dV = \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$.
(12)

1. In the following picture, label ρ, θ, and ϕ, and calculate r, x, y, and z in terms of ρ, θ, and ϕ.



2. Write an integral in spherical coordinates which would calculate the mass of the solid hemisphere of radius 3, given by the equations $x^2 + y^2 + z^2 = 9$ and $z \geq 0$, if the density is proportional to the distance to the z-axis, with k as the constant of proportionality. Supply limits of integration, but *do not* continue with the calculation of the integral.

3. For the same solid as in the previous part, write an integral in spherical coordinates which would calculate the moment with respect to the xy-plane. Supply limits of integration, but *do not* continue with the calculation of the integral.

VI. Consider the surface which is in the upper hemisphere $x^2 + y^2 + z^2 = 4$ and $z \geq 0$, and lies inside the
(13) cylinder $x^2 + y^2 = 1$.

1. Write an integral in xy-coordinates whose value is the area of the surface. Supply limits of integration, but *do not* go on to calculate the value of the integral.

2. Change the integral to polar coordinates, and supply limits of integration.

3. Calculate the integral in polar coordinates, to find the surface area.