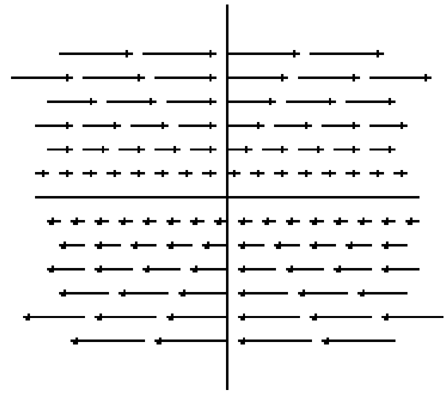
Mathematics 2443-002 Name (please print) ____________________

Examination III Form A

December 1, 2000

I. (4.5) The figure to the right shows a vector field $\vec{F}$ in the xy-plane.



1. If C is the line segment from $(1,0)$ to $(1,2)$, is $\int_C \vec{F}\cdot d\vec{r}$ positive, negative, or 0?

2. If C is the line segment from $(0,1)$ to $(2,1)$, is $\int_C \vec{F}\cdot d\vec{r}$ positive, negative, or 0?

3. If C is the unit circle, oriented counterclockwise, is $\int_C \vec{F}\cdot d\vec{r}$ positive, negative, or 0?

II. (12) A curve C is parameterized as $\vec{r}(t) = t^2\vec{\imath} + \sin(t)\vec{\jmath}$ for $0 \leq t \leq \pi$. For each of the following, use the parameterization to express the line integral as an ordinary definite integral of a function of t, but *do not* try to calculate the actual values of the integrals.

1. $\int_C x^2y\,ds$

2. $\int_C x^2y\,dx$

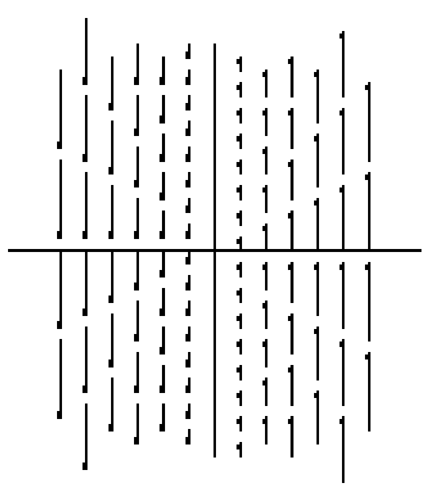
3. $\int_C (x^2\vec{\imath} + y\vec{\jmath})\cdot d\vec{r}$

4. For a wire bent in the shape of C, with density $\rho(x,y) = x+3$, the moment of the wire with respect to the y-axis

III. (5) Calculate $\operatorname{curl}(xy\vec{\imath} + yz\vec{\jmath} + zx\vec{k})$.

IV. (5) Let C be the portion of the graph of $y = \ln(x)$ that runs from $(e, 1)$ to $(e^2, 2)$, and let $\vec{F}$ be the gradient of the function $f(x, y) = \dfrac{\ln(x)y^2}{x}$. Calculate $\int_C \vec{F} \cdot d\vec{r}$. (*Recommendation:* If this problem is taking you more than 2 minutes, go on to other problems and return to this one if you have time.)

V. (4.5) A vector field $P\vec{\imath} + Q\vec{\jmath}$ is shown to the right (on the y-axis, it consists of the zero vector at each point).

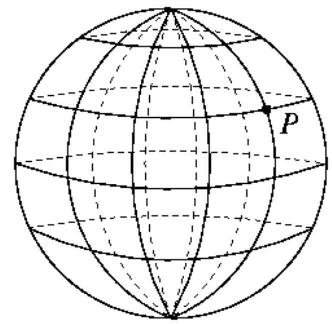


1. Tell whether $\dfrac{\partial Q}{\partial x}$ is positive, negative, or zero.

2. Tell whether $\dfrac{\partial Q}{\partial y}$ is positive, negative, or zero.

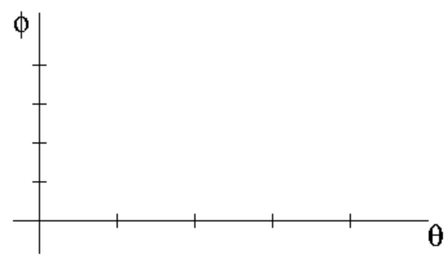
3. Tell whether $\dfrac{\partial P}{\partial x}$ is positive, negative, or zero.

VI. (10) The figure to the right shows the sphere S of radius a, and a point P which lies in the front part of S (the coordinates of P are approximately $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$). Let ϕ and θ be the usual spherical coordinates. They define parameters on S.



1. Give x, y, and z in terms of ϕ and θ.

2. Tell the parameter domain R mathematically (that is, in terms of inequalities involving ϕ and θ), and draw it in the $\theta\phi$-plane shown at the right (label the coordinate axes correctly).



3. At the point P, indicate the curve on S where ϕ is constant, and the curve where θ is constant. Draw and label the vectors $\vec{r}_\theta$ and $\vec{r}_\phi$ at the point P, and draw an outward normal vector at P.

4. Is the outward normal vector $\vec{r}_\theta \times \vec{r}_\phi$, or is it $\vec{r}_\phi \times \vec{r}_\theta$?

VII. Let D be the domain in the plane consisting of all (x, y) with $y > 0$. It is simply-connected.
(6)

1. Use partial derivatives to verify that the vector field $(\ln(y) - x)\vec{\imath} + \left(\frac{x}{y} - y\right)\vec{\jmath}$ is conservative.

2. Find a function $f(x, y)$ with $\nabla f = (\ln(y) - x)\vec{\imath} + \left(\frac{x}{y} - y\right)\vec{\jmath}$

VIII. Let S be the rectangle in the plane with vertices the points $(2, 0)$, $(3, 0)$, $(2, 3)$, and $(3, 3)$. Let C be the
(6) boundary of S, with the positive orientation. For each of the following, use a version of Green's Theorem to carry out a very easy calculation of the given integral. (*Recommendation:* If this problem is taking you more than 2 minutes, work on other problems and return to this one if you have time.)

1. $\int_C (xy^4 + 3y)\, dx + (2x^2y^3 - 2x)\, dy$.

2. $\int_C \left((2x^2y^3 + x)\vec{\imath} + (3y - xy^4)\vec{\jmath}\right) \cdot \vec{n}\; ds$, where $\vec{n}$ is the unit outward normal of C.