

I. Rewrite the following integrals using polar or spherical coordinates. Be sure to supply the correct limits of integration for the new variables. *Do not* carry out the calculation of the integrals.

1. $\int_0^1 \int_{-\sqrt{1-x^2}}^0 e^{x^2+y^2} dy dx$

2. $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2} dz dy dx$

II. A curve C is parameterized as $\vec{r}(t) = e^t \vec{i} + \ln(\cos(t)) \vec{j}$ for $0 \leq t \leq 1$. Using this parameterization, express each of the following as an ordinary definite integral of a function of t . Make reasonable algebraic simplifications, but *do not* try to calculate the actual values of the integrals.

1. $\int_C \ln(x)e^y ds$

2. $\int_C e^x dy$

3. $\int_C (x^2 \vec{i} + e^y \vec{j}) \cdot d\vec{r}$

- III. Let $\vec{F}(x, y, z) = -xz\vec{i} - yz\vec{j} + z^2\vec{k}$, and let the surface S be the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Calculate
 (4) $\iint_S \vec{F} \cdot d\vec{S}$, where S has the outward normal.

- IV. Let r and θ be the polar coordinates in the xy -plane. For the function $f(x, y) = x^3y$, calculate $\frac{\partial f}{\partial \theta}$ in terms of x and y .
 (5)

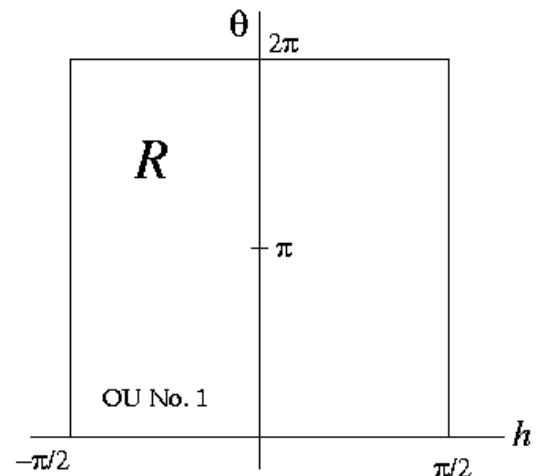
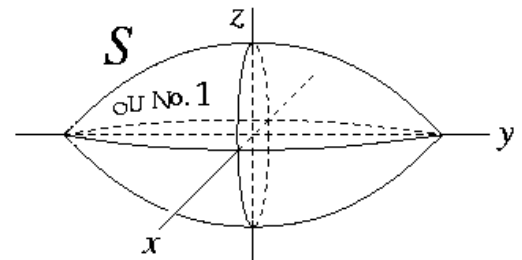
- V. The picture at the right shows a parameterization of the outer surface S of a football. It is parameterized by letting θ be the polar angle in the xz -plane, and h be the y -coordinate. The parameterization is

$$\begin{aligned} x &= \cos(h) \cos(\theta) \\ y &= h \\ z &= \cos(h) \sin(\theta), \end{aligned}$$

where the parameter domain R in the $h\theta$ -plane consists of $\frac{\pi}{2} \leq h \leq \frac{3\pi}{2}$, $0 \leq \theta \leq 2\pi$.

1. Calculate \vec{r}_h and \vec{r}_θ .

2. For the line $0 \leq h \leq \frac{\pi}{2}$, $\theta = \frac{\pi}{4}$ in R , draw the corresponding points on \vec{S} . Do the same for the line $h = \frac{\pi}{4}$, $0 \leq \theta \leq \frac{\pi}{2}$. At the intersection point of these two curves on S , draw the vectors \vec{r}_θ and \vec{r}_h .



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3. Given that $\vec{r}_h \times \vec{r}_\theta = \cos(h) \cos(\theta) \vec{i} + \sin(h) \cos(h) \vec{j} + \cos(h) \sin(\theta) \vec{k}$, carry out the following:

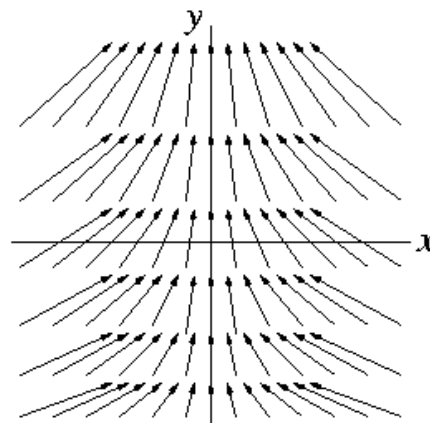
(a) Check that $\|\vec{r}_h \times \vec{r}_\theta\| = \cos(h) \sqrt{1 + \sin^2(h)}$.

(b) Calculate $\iint_S \sqrt{1 + \sin^2(y)} dS$.

(c) Calculate $\iint_S \sin(y) \cos(y) \vec{j} \cdot d\vec{S}$, where the outward normal is used on S .

(d) Given that the volume of the football is $\frac{\pi^2}{2}$, calculate the flux of the vector field $(x+y)\vec{i} + (y+z)\vec{j} + (z+x)\vec{k}$ across S , where the outward normal is used on S .

VI. The picture at the right shows a certain vector field $P\vec{i} + Q\vec{j}$ in the xy -plane.



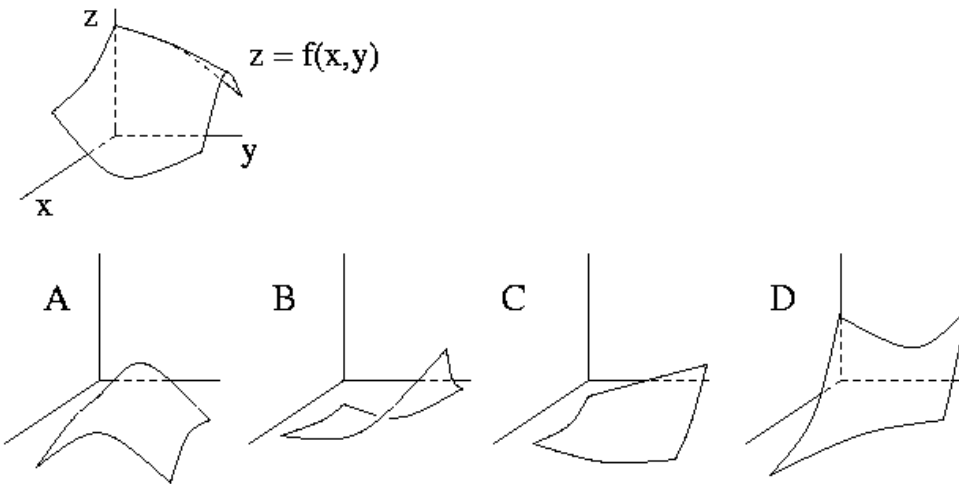
1. Is $\frac{\partial P}{\partial x}$ positive, negative, or zero?
2. Is $\frac{\partial P}{\partial y}$ positive, negative, or zero?
3. Is $\frac{\partial Q}{\partial x}$ positive, negative, or zero?
4. Is $\frac{\partial Q}{\partial y}$ positive, negative, or zero?
5. If C is the line segment from $(-1, 1)$ to $(0, 1)$, is $\int_C (P\vec{i} + Q\vec{j}) \cdot d\vec{r}$ positive, negative, or zero?
6. If C is the line segment from $(0, 1)$ to $(1, 1)$, is $\int_C (P\vec{i} + Q\vec{j}) \cdot d\vec{r}$ positive, negative, or zero?

VII. Let $f(x, y, z)$ be the function xe^{x+y+2z} .

- (8)
 1. Calculate the gradient $\nabla f(x, y, z)$.
 2. Calculate the rate of change of f at the point $(3, 1, 1)$, in the direction toward the point $(0, 0, 2)$.
 3. Let S be the level surface of f through the point $(3, 1, 1)$. Find an equation for the tangent plane to S at the point $(3, 1, 1)$.

- VIII.** Let C consist of the clockwise arc of the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(0, 2)$, followed by the line segment from $(0, 2)$ to $(2, 4)$. Calculate $\int_C (2xy \cos(x^2) \vec{i} + (\sin(x^2) + \cos(y)) \vec{j}) \cdot d\vec{r}$.

- IX.** The following two questions refer to the function $f(x, y)$ whose graph is shown here, and the four functions indicated in the graphs A, B, C, and D. All of the functions have domain the points (x, y) with $0 \leq x \leq 1$ and $0 \leq y \leq 1$.



- The graph of the function $\frac{\partial f}{\partial x}$ looks most like the function in
 - A
 - B
 - C
 - D
- The graph of the function $\frac{\partial f}{\partial y}$ looks most like the function in
 - A
 - B
 - C
 - D

X. Let T be the part of the plane $x + y + z = 1$ that lies in the first octant, with the upward normal, and let
(10) C be its boundary, with the positive orientation.

1. Use Stokes' Theorem to write $\int_C (xz\vec{i} + 3xy\vec{j} + 3xy\vec{k}) \cdot d\vec{r}$ as a surface integral on T .

2. Regarding T as the graph of the function $z = 1 - x - y$ over a domain D in the xy -plane, use the formula $\iint_S (P\vec{i} + Q\vec{j} + R\vec{k}) \cdot d\vec{S} = \iint_D -P g_x - Q g_y + R dD$ to calculate the surface integral.

XI. Let D be a domain in the plane. Define what it means to say that D is *simply-connected*.
(3)