Mathematics 2433-003H — Name (please print) ____________________

Examination I

September 25, 2001

Instructions: Give brief, clear answers. If needed, take as known the convergence behavior of the geometic series $\sum r^n$ and the p-series $\sum \frac{1}{n^p}$.

I. (6) Use the Integral Test to check that $\sum \frac{1}{n}$ diverges. Draw a picture showing the underlying geometric reason for the divergence.

II. (6) State the Monotonicity Theorem for sequences. Why is it a theorem not so much about sequences as about the real numbers?

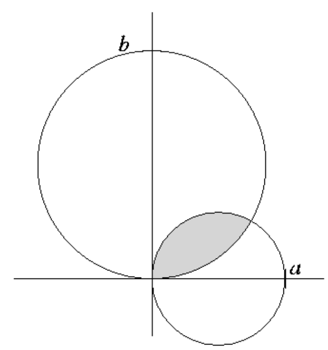
III. (8) Use the Alternating Series Test to show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^n}$ converges. Use the Ratio Test and the limit $\displaystyle\lim_{x\to\infty} (1+\tfrac{c}{x})^x = e^c$ to show that it converges absolutely.

IV. (5) Use the formula $ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ to calculate ds if $r^2 = \cos(2\theta)$. Simplify the resulting expression.

V. (8) Give examples of:

1. A divergent series of positive terms $\sum a_n$ for which $\sum a_n^2$ is convergent.
2. A convergent series $\sum a_n$ such that $\sum a_n^2$ is divergent.

VI. (5) Write a sum of two definite integrals in polar coordinates whose value is the area of the shaded region in the diagram to the right, but *do not* evaluate the integrals or proceed further with finding the area.



VII. (5) Calculate $\displaystyle\sum_{n=0}^{\infty} \cos^{2n}(\theta)$ (the answer is $\csc^2(\theta)$).

VIII. (5) Describe the motion of a particle P that moves according to the parametric equations $x = \cos(\cos(t))$, $y = \sin(\cos(t))$.

IX. (12) For each of the following series, determine whether the series is convergent or divergent, and give an explanation of your reasoning.

1. $\displaystyle\sum_{k=1}^{\infty} k^{-2.3}$
2. $\displaystyle\sum_{n=1}^{\infty} a_n^2$, given that $\sum a_n$ is absolutely convergent (hint: for large n, $|a_n| < 1$)
3. $\displaystyle\sum_{n=1}^{\infty} \ln(a_n)$, given that $\sum a_n$ is a convergent series with positive terms