Mathematics 2443-006 Name (please print) _____

Examination I

September 20, 2001

I. Calculate the following.

(15)

1. The maximum rate of change of the function $f(x, y) = \ln(x^2 + y^4)$ at the point $(2, 1)$, and the direction in which it occurs.

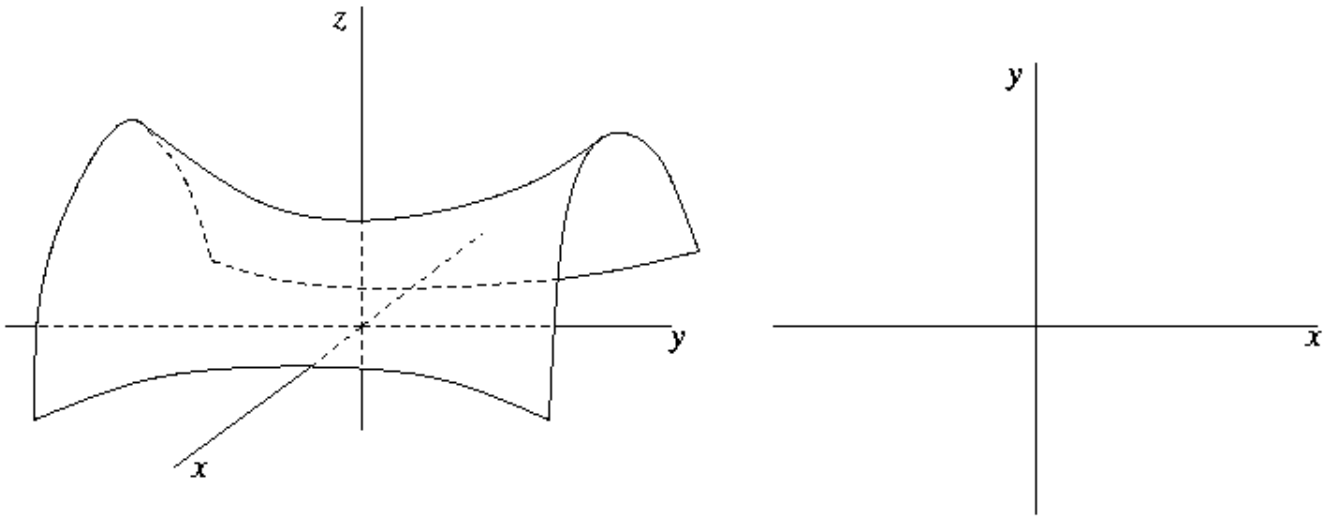
2. $\dfrac{\partial x}{\partial f}$ if $x(f, g) = e^{fg}$.

3. $\dfrac{\partial z}{\partial x}$ if $xy = \sin(z^2)$.

4. $\dfrac{\partial f}{\partial x_5}$ if $f(x_1, x_2, \ldots, x_n) = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$.

5. dz if $z = x^2 + \ln(y^2)$

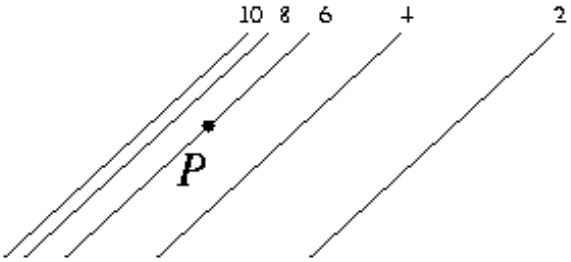
II. (3) In the figure below, the xyz-coordinate system on the left shows the graph of a certain function of two variables. The portion shown here has $z \geq 0$. In the xy-coordinate system on the right, sketch some level curves for the function, including the one through $(0, 0)$.



III. (4) Write the chain rule for $\dfrac{\partial a}{\partial b}$ if $a = a(x, y, z, w)$, $x = x(b, c)$, $y = y(b, c)$, $z = z(b, c)$, and $w = w(b, c)$.

IV. (4) Find the limit, if it exists, or show that the limit does not exist: $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$.

V. (6) Some level lines of a certain function $g(x, y)$ near a point P are shown to the right. Answer the following, assuming the most likely behavior of g indicated by the values of g on these level lines.

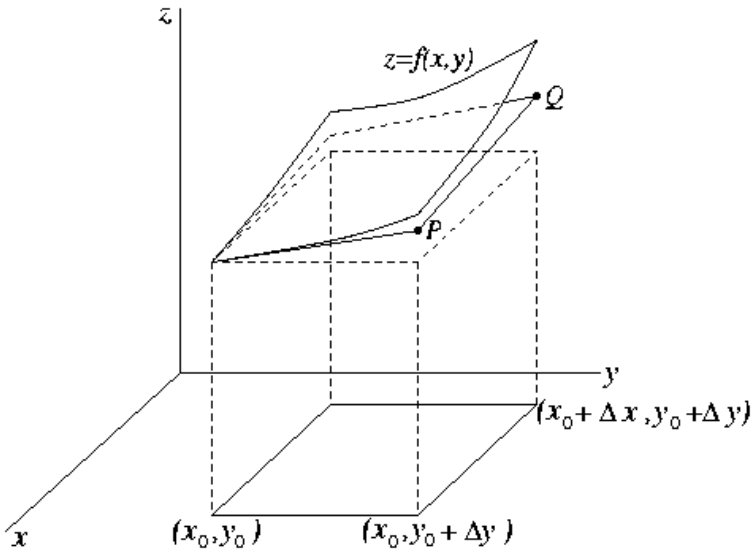


1. Is $\dfrac{\partial g}{\partial x}$ is positive, negative, or zero?

2. Is $\dfrac{\partial^2 g}{\partial y^2}$ is positive, negative, or zero?

3. Draw ∇g at P.

VI. (4) The figure to the right shows the graph of a function $f(x, y)$ near a certain point (x_0, y_0), and the tangent plane to the graph of f at the point $(x_0, y_0, f(x_0, y_0))$. The points P and Q lie on the tangent plane. Suppose that $f(x_0, y_0) = 2$, $f_x(x_0, y_0) = -0.2$ and $f_y(x_0, y_0) = 0.3$. In terms of Δx and Δy, find the z-coordinate of P and the z-coordinate of Q.



The z-coordinate of P is:

The z-coordinate of Q is:

VII. Calculate the following.
(16)

1. The directional derivative of g at $(1, 2)$ in the direction toward $(0, 3)$, if $\nabla g(x, y) = 4xy^2\,\vec{\imath} + 4x^2y\,\vec{\jmath}$.

2. All critical points of the function $f(x, y) = xy - 2x - y$.

3. The absolute maximum and the absolute minimum of the function $h(x, y) = x^2 + y^2 + x^2y$ on the boundary of the square $D = \{(x, y) \mid |x| \leq 1,\ |y| \leq 1\}$.

4. The number c if for a certain function $f(x, y)$, $\dfrac{\partial f}{\partial x} = 5xy + \dfrac{1}{\sqrt{1 - \sin(x^3)}}$ and $\dfrac{\partial f}{\partial y} = 8\,\tan^{-1}(y) + cx^2$.