Mathematics 2443-006 Name (please print) ____________________

Examination II

October 23, 2001

I. (12) For each of the following, write a double integral whose value is the quantity described in the problem. Supply limits of integration, in the appropriate coordinates for the domain, but *do not* calculate the value of the integral.

1. The area of one loop of the rose $r = \cos(3\theta)$.

2. The mass of a lamina that occupies the region in the xy-plane with $x \geq 0$, $y \geq 0$, and $1 \leq x^2 + y^2 \leq 2$, if the density is twice the distance from the origin.

3. The x-coordinate of the center of mass of the lamina in the previous problem.

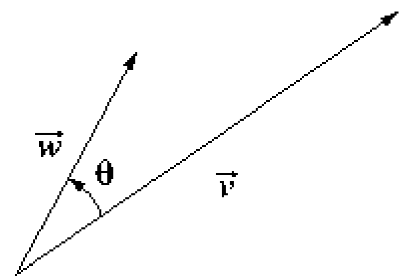
4. The volume under the graph of the function $\sin^2(xy)$ and over the triangle with vertices $(0, 0)$, $(1, 1)$, and $(2, 0)$.

II. (4) A table of values is given for a function $f(x,y)$ defined on $R = [0,4] \times [1,9]$:

$x =$ \ $y =$	3	5	7
1	4	-3	7
3	3	1	2

Estimate $\iint_R f(x,y)\, dA$ using the midpoint rule with $m = n = 2$ (that is, calculate the numerical values of the Riemann sum where the x-interval is partitioned into two equal intervals and the y-interval is partitioned into two equal intervals, and the value of the function is taken at the center of each rectangle).

III. (4) The picture to the right shows two vectors $\vec{v}$ and $\vec{w}$, and the angle θ between them. Assume that $\|\vec{v}\| = 5$, $\|\vec{w}\| = 3$, and $\vec{v} \cdot \vec{w} = 9$.



1. Use the geometric meaning of the dot product to calculate $\cos(\theta)$.
2. Use $\cos(\theta)$ to calculate $\sin(\theta)$.
3. Calculate the area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$.

IV. (4) Consider an integral $\iint_R f(x,y)\, dA$, where R is the region which lies between the graphs of $y = \cos(x)$ and $y = -\cos(x)$ and between the lines $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. Supply limits of integration for this integral.

V. Evaluate each of the following multiple integrals.
(15)

1. $\displaystyle\iint_R \frac{x}{1+y^2}\, dA$ where R is the rectangle $0 \le x \le 5$ and $0 \le y \le 1$.

2. $\iint_R \dfrac{1}{1+x^2+y^2}\, dA$, where R is the region in the first quadrant between the circles $x^2+y^2 = 1$ and $x^2+y^2 = 3$.

3. $\int_0^1 \int_{2y}^2 e^{x^2}\, dx\, dy$ (Hint: Change the order of integration.)

VI. Calculate the surface area of the portion of $z = 2 - x^2 - y^2$ that lies above the xy-plane.
(6)

VII. Let E be upper half of the solid ball of radius 1, that is, the points with $x^2+y^2+z^2 \leq 1$, $z \geq 0$. Use spherical
(6) coordinates to calculate $\iiint_E z\,dV$. (Recall that $x = \rho\cos(\theta)\sin(\phi)$, $y = \rho\sin(\theta)\sin(\phi)$, $z = \rho\cos(\phi)$, and $dV = \rho^2\sin(\phi)\,d\phi\,d\theta\,dz$.)