Math 5853 homework

Instructions: All problems should be prepared for presentation at the problem sessions. If a problem has a due date listed, then it should be written up formally and turned in on the due date.

65. (1/18) Two surfaces F_1 and F_2 can be constructed as follows. Start with $S^1 \times I$, and remove the interior of a small disk D from the interior of $S^1 \times I$. For F_1, identify each $(\theta, 0)$ with $(\theta, 1)$ and identify each point of ∂D with its antipodal point (that is, if ∂D is regarded as S^1, then v is identified with $-v$). For F_2, identify each $(\theta, 0)$ with $(\overline{\theta}, 1)$ and identify each point of ∂D with its antipodal point.

    1. Make drawings illustrating each of F_1 and F_2. Notice that both are closed surfaces.
    2. Find three disjoint Möbius bands imbedded in F_1.
    3. Find three disjoint Möbius bands imbedded in F_2.

    Actually, F_1 and F_2 are homeomorphic, although this may not be very easy to see.

66. (1/18) Let M and N be n-dimensional manifolds, and let U be an open subset of M. Suppose that $f\colon U \to N$ is a continuous injection. Prove that f takes open sets in U to open sets in N.