Mathematics 2433-001H Name (please print) ____________________

Examination II

October 24, 2005

I. (20) For each of the following series, use standard facts and/or convergence tests to determine whether the series converges or diverges. Give only brief details, but indicate clearly what fact or test you are using, and give at least the key steps in verifying that the test applies.

(i) $\sum \dfrac{5^{n+2}}{3^{2n}}$

(ii) $\sum \dfrac{(-1)^n}{2^{1/n}}$

(iii) $\sum \tan\left(\dfrac{1}{n}\right)$

(iv) $\sum \dfrac{1}{n + n\cos^2(n)}$

(v) $\sum \dfrac{n^4 - 7n^3 + 1}{n^7 - n^4 + 13n}$

II. (4) A power series of the form $\sum c_n(x-\pi)^n$ converges at $x = 2$ and diverges at $x = 10$. From this information, what can be determined about its radius of convergence?

III. (8) Find the radius of convergence and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x+3)^n$.

IV. (6) Fill in the missing parts of the following argument: Suppose that $\sum |a_n|$ converges. Since $0 \le a_n + |a_n| \le 2|a_n|$, the Comparison Test shows that [fill in]. Since $\sum(a_n + |a_n|)$ and $\sum$ [fill in] converge, it follows that $\sum$ [fill in] converges.

V. (5) Let $\sum a_n$ be a series with all $a_n > 0$, and assume that $\lim a_n = 0$. Use the Limit Comparison Test to show that $\sum a_n$ converges if and only if $\sum \ln(1 + a_n)$ converges.

VI. (9) Let $\{a_n\}$ be a sequence. The infinite product $\displaystyle\prod_{n=1}^{\infty} a_n$ is (not surprisingly) defined to be $\displaystyle\lim_{n\to\infty} p_n$ where p_n is the *partial product* defined by $p_n = \displaystyle\prod_{i=1}^{n} a_i$.

1. Calculate $\displaystyle\prod_{n=1}^{\infty} \frac{n}{n+1}$.
2. Show that $\displaystyle\prod_{n=1}^{\infty} 2^{\frac{1}{n^\alpha}}$ converges if and only if $\alpha > 1$.
3. Assuming that all $a_n > 0$, show that $\displaystyle\prod_{n=1}^{\infty}(1 + a_n)$ converges if and only if $\sum a_n$ converges.