Mathematics 2433-001H Name (please print) ____________________

Examination I

September 20, 2007

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (4) A curve is give parametrically by the equations $x = \int_0^t \cos(\pi u^2/2)\, du$, $y = \int_0^t \sin(\pi u^2/2)\, du$. Find the length of the portion of this curve with $0 \le t \le \pi$.

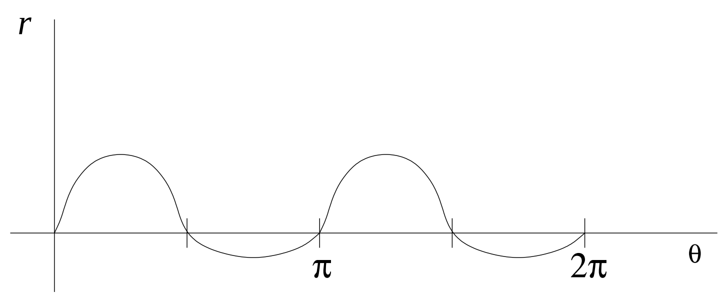
II. (4) An equation $r = f(\theta)$ defines a polar curve. Use the Chain Rule $\dfrac{dy}{dx} = \dfrac{\frac{dy}{du}}{\frac{dx}{du}}$ to derive a general formula for $\dfrac{dy}{dx}$ in terms of r and θ for such a curve.

III. (4) A curve given by the parametric equations $x = 2t^3$, $y = 1 - t^2$, $-\infty < t < \infty$. Find the area of the region bounded by the curve and the x-axis.

IV. (4) Find the surface area of a sphere of radius R by regarding it as $x = R\cos(\theta)$, $y = R\sin(\theta)$ and rotating about the x-axis.

V. (4) Calculate the area of the region that lies inside the polar curve $r = 4\sin(\theta)$ and outside the polar curve $r = 2$.

VI. (4) The graph of a certain equation $r = f(\theta)$ is shown at the right, in a rectangular θ-r coordinate system. In an x-y coordinate system, make a reasonably accurate graph of the polar equation $r = f(\theta)$ for this function.



VII. (5) State the Squeeze Theorem. Use the Squeeze Theorem to find the limit of $\left\{ \dfrac{(2n-1)!}{(2n+1)!} \right\}$ by comparing it to the sequence $\{0\}$ and to some sequence of the form $\{n^p\}$.

VIII. (4) Determine whether each of the following series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \arctan(n)$

2. $\displaystyle\sum_{n=1}^{\infty} (\sin(1))^n$

IX. (4) Find all x for which the series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{x^n}$ converges.

X. (8) State the Monotonicity Theorem. Analyze the convergence of the sequence $\left\{\dfrac{n}{n^2+1}\right\}$ as follows:

1. State the Monotonicity Theorem.
2. Calculate that the derivative of the function $\dfrac{x}{x^2+1}$ is nonpositive when $x \geq 1$. Deduce that $\left\{\dfrac{n}{n^2+1}\right\}$ is decreasing.
3. Verify any other hypotheses of the Monotonicity Theorem, to deduce that $\left\{\dfrac{n}{n^2+1}\right\}$ converges.
4. Now, find the limit by dividing numerator and denominator by n and observing the effect of letting $n \to \infty$.

XI. (5) Use a simple diagram involving dr and $d\theta$ to derive an expresssion for ds in terms of dr and $d\theta$.