Mathematics 2433-001H Name (please print) ______________________

Examination III

November 15, 2007

I. (4) Find an equation for the sphere that has center $(4, -2, \pi)$ and contains the origin.

II. (4) Find parametric equations for the line that is the intersection of the planes $x-2y-3z=1$ and $2x+y+z=1$.

III. (6) Determine the convergence or divergence of each of these series, using any information or method *other than the Limit Comparison Test.*

1. $\displaystyle\sum_{n=1}^{\infty} \frac{7+7^n}{8+8^n}$

2. $\displaystyle\sum_{n=1}^{\infty} \Big(\frac{n-1}{n}\Big)^{n^2}$

IV. (6) Carry out a translation of the form $X = x - x_0$, $Y = y - y_0$, $Z = z - z_0$ to put the equation $x^2 - y^2 - 4z^2 = 2x + 16z + 16$ into standard form. (You do not have to draw the graph, but you can if you want.) The graph is a hyperboloid of two sheets. Which traces are ellipses? At what points (in XYZ-coordinates) does it meet the X-axis?

V. (4) Let $\vec{a} = -4\vec{\imath} + \vec{\jmath} + 3\vec{k}$ and $\vec{b} = \vec{\imath} + \vec{\jmath} - \vec{k}$.

1. Calculate the scalar projection of $\vec{a}$ onto $\vec{b}$.
2. Calculate the vector projection of $\vec{a}$ onto $\vec{b}$.

VI. (4) Give an algebraic verification that $\|\,\vec{a}+\vec{b}\,\|^2 + \|\,\vec{a}-\vec{b}\,\|^2 = 2\,\|\,\vec{a}\,\|^2 + 2\,\|\,\vec{b}\,\|^2$, but not by doing a lengthy calculation involving $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$.

VII. (4) Give examples of the following:

1. Vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ for which $(\vec{a}\times\vec{b})\times\vec{c} \neq \vec{a}\times(\vec{b}\times\vec{c})$.
2. Nonzero vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ for which $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$ but $\vec{b}\neq\vec{c}$.

VIII. (4) Here is a fact: Let $\vec{u}$ be any unit vector (i. e. $\|\,\vec{u}\,\| = 1$). If $\vec{v}$ is any vector, then the length of $\vec{v}\times\vec{u}$ is no more than the length of $\vec{v}$.

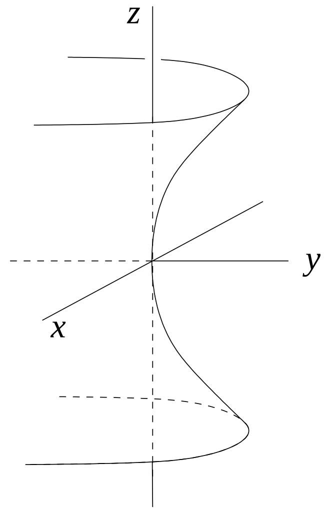
1. Give an algebraic explanation for this fact.
2. Give a geometric explanation for this fact.

IX. (4) If you saw a movie of the planes $y\cos(\theta) + z\sin(\theta) = 0$ as θ went from 0 to 2π, what would they look like? Use both words and picture(s) in your explanation.

X. (4) Derive these formulas expressing rectangular coordinates in terms of spherical coordinates: $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$.

XI. (4) Draw the graph of this equation given in spherical coordinates: $\rho = \phi^3$.

XII. Write a possible equation for this saddle surface:
(4)



XIII. In higher dimensions, say dimension n, there are vectors $\vec{e}_1$, $\vec{e}_2, \ldots,$ $\vec{e}_n$ that play the roles of $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$. In
(4) particular, $\vec{e}_i \cdot \vec{e}_j = 0$ when $i \neq j$, and $\vec{e}_i \cdot \vec{e}_i = 1$ for each i. Verify that if an n-dimesional vector $\vec{v}$ equals $r_1\vec{e_1} + r_2\vec{e_2} + \cdots + r_n\vec{e_n}$, then $r_1 = \vec{v} \cdot \vec{e}_1$.