Mathematics 1823-030 Examination II Form A October 19, 2009 (1) **Discussion Section** (circle day and time): Th 9:00 Th 1:30 Th 3:00 F 8:30 F 9:30 F 2:30

I. The figure to the right shows the graph of a certain func-(6) tion  $f: [-2, 4] \to \mathbb{R}$ . On the coordinate system shown below, sketch a graph of the derivative f'(x). The values of f'(x) need not be precise, but should accurately reflect the behavior of f(x). Indicate clearly any points where f'(x) is underfined.



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II. Find the value of  $\lim_{\theta \to 0} \frac{\sin(5\theta)}{\sin(7\theta)}$  (not by plotting points or using l'Hôpital's rule).

III. Calculate each of the following. When calculating derivatives, use the algebraic rules, the Chain Rule,
(25) and/or implicit differentiation (i. e. do not use the definition of the derivative as a limit). Do not perform simplifications to the answer unless instructed to do so.

(i) 
$$\frac{dy}{dx}$$
 if  $y = \frac{\sin(x^2)}{\sin^2(x)}$ 

(ii) 
$$\frac{d^2y}{dx^2}$$
 if  $y = \cot(x)$ 

(iii) An equation for the tangent line to the curve  $y = (x^2 - 1)^{1,000}$  at the point  $(\sqrt{2}, 1)$ . You need not simplify the answer.

(iv) 
$$\frac{dw}{dt}$$
 if  $w = \sqrt{\frac{t-1}{t+1}}$ . Simplify this answer.

(v) 
$$\frac{dy}{dx}$$
 if  $x^2 \cos(y) = y^3$ 

**IV**. Define what it means to say that a function f is *continuous at*  $x_0$ . State the Intermediate Value Theorem. (5)

V. State the precise, formal (i. e. using  $\epsilon$  and  $\delta$ ) definition of:  $\lim_{x \to 3\pi/4} \cot(x) = -1$ . (3)

**VI**. Let f be a function which is differentiable at x = a. Label each of the following statements either T for (6) true or F for false.

\_\_\_\_\_ The limit  $\lim_{w \to a} \frac{f(w) - f(a)}{w - a}$  must exist.

f must be differentiable on any open interval that contains a.

\_\_\_\_\_ f must be continuous at x = a.

f'(a) might be  $\infty$  or  $-\infty$ .

VII. In the blank to the left of each of the following two questions, write the letter of the best response. (4)

1. \_\_\_\_\_ What type of mathematical object is  $\frac{d^2y}{dx^2}$ ? A) set B) function C) equation D) codomain E) number F) theorem

2 The	e pair $(4!, 0!)$ equ	als		
A) $(4, 0)$	B) $(12, 0)$	C) $(20, 0)$	D) $(24, 0)$	E) $(32, 0)$
G) $(4, 1)$	H) $(12, 1)$	I) $(20, 1)$	J) $(24, 1)$	K) $(32, 1)$

VIII. The table to the right shows the values of the func-(6) tions f, g, f', and g' at the x-values 1, 2, 3, and 4. For example, f(4) = 2 and f'(4) = 4. Write the value of each of the following:

$(g \cdot f)'(3) = \underline{\qquad}$	$(g \circ f)'(1) = \underline{\qquad}$	$(f/g)'(3) = \underline{\qquad}$	$(f \circ f)(4) = \underline{\qquad}$

x	f(x)	f'(x)	g(x)	g'(x)
1	3	2	4	1
2	2	3	1	4
3	4	1	4	2
4	2	4	1	2

F) (120, 0)

L) (120, 1)