Mathematics 1823-030
Examination II Form A
October 19, 2009

Name (please print)
Student Number
(1) Discussion Section (circle day and time):

Th 9:00 Th 1:30 Th 3:00 F 8:30 F 9:30 F 2:30
I. The figure to the right shows the graph of a certain func-
(6) tion $f:[-2,4] \rightarrow \mathbb{R}$. On the coordinate system shown below, sketch a graph of the derivative $f^{\prime}(x)$. The values of $f^{\prime}(x)$ need not be precise, but should accurately reflect the behavior of $f(x)$. Indicate clearly any points where $f^{\prime}(x)$ is underfined.

II. Find the value of $\lim _{\theta \rightarrow 0} \frac{\sin (5 \theta)}{\sin (7 \theta)}$ (not by plotting points or using l'Hôpital's rule).
III. Calculate each of the following. When calculating derivatives, use the algebraic rules, the Chain Rule, (25) and/or implicit differentiation (i. e. do not use the definition of the derivative as a limit). Do not perform simplifications to the answer unless instructed to do so.
(i) $\frac{d y}{d x}$ if $y=\frac{\sin \left(x^{2}\right)}{\sin ^{2}(x)}$
(ii) $\frac{d^{2} y}{d x^{2}}$ if $y=\cot (x)$
(iii) An equation for the tangent line to the curve $y=\left(x^{2}-1\right)^{1,000}$ at the point $(\sqrt{2}, 1)$. You need not simplify the answer.
(iv) $\frac{d w}{d t}$ if $w=\sqrt{\frac{t-1}{t+1}}$. Simplify this answer.
(v) $\frac{d y}{d x}$ if $x^{2} \cos (y)=y^{3}$
IV. Define what it means to say that a function $f$ is continuous at $x_{0}$. State the Intermediate Value Theorem. (5)
V. State the precise, formal (i. e. using $\epsilon$ and $\delta$ ) definition of: $\lim _{x \rightarrow 3 \pi / 4} \cot (x)=-1$.
VI. Let $f$ be a function which is differentiable at $x=a$. Label each of the following statements either $T$ for (6) true or $F$ for false.
$\ldots$ The limit $\lim _{w \rightarrow a} \frac{f(w)-f(a)}{w-a}$ must exist.
$\ldots \quad f$ must be differentiable on any open interval that contains $a$.
$\ldots$ must be continuous at $x=a$.
$\ldots f^{\prime}(a)$ might be $\infty$ or $-\infty$.
VII. In the blank to the left of each of the following two questions, write the letter of the best response.
(4)

1. What type of mathematical object is $\frac{d^{2} y}{d x^{2}}$ ?
A) set
B) function
C) equation
D) codomain
E) number
F) theorem
2. $\qquad$ The pair (4!, 0!) equals
A) $(4,0)$
B) $(12,0)$
C) $(20,0)$
D) $(24,0)$
E) $(32,0)$
F) $(120,0)$
G) $(4,1)$
H) $(12,1)$
I) $(20,1)$
J) $(24,1)$
K) $(32,1)$
L) $(120,1)$
VIII. The table to the right shows the values of the functions $f, g, f^{\prime}$, and $g^{\prime}$ at the $x$-values $1,2,3$, and 4. For example, $f(4)=2$ and $f^{\prime}(4)=4$. Write the value of each of the following:

| $x$ | $f(x)$ | $f^{\prime}(x)$ | $g(x)$ | $g^{\prime}(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 3 | 2 | 4 | 1 |
| 2 | 2 | 3 | 1 | 4 |
| 3 | 4 | 1 | 4 | 2 |
| 4 | 2 | 4 | 1 | 2 |

$(g \cdot f)^{\prime}(3)=$ $\qquad$ $(g \circ f)^{\prime}(1)=$ $\qquad$ $(f / g)^{\prime}(3)=$ $\qquad$

$$
(f \circ f)(4)=
$$

