Mathematics 1823-030
Examination III Form A
November 23, 2009

Name (please print) ______________________

Student Number ______________

(1) **Discussion Section** (circle day and time):

Th 9:00 Th 1:30 Th 3:00 F 8:30 F 9:30 F 2:30

I. (10) In this problem, $f(x) = \frac{2}{x} + \frac{1}{x^2}$, which can also be written as $\frac{2}{x^2}\left(x + \frac{1}{2}\right)$. The first and second derivatives of f are $f'(x) = -\frac{2}{x^2} - \frac{2}{x^3}$ and $f''(x) = \frac{4}{x^3} + \frac{6}{x^4}$ (do not check these)

1. Find the root or roots of f.

2. Find $\lim_{x\to\infty} f(x)$.

3. Find the critical number or critical numbers of f.

4. Find the inflection point or inflection points of f.

5. Find $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 0^-} f(x)$.

II. (7) (a) Find the differential of $\sqrt{x}$.

(b) Use linear approximation to estimate $\sqrt{8.994}$.

III. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on
(8) the dock that is 3 m higher than the bow of the boat. The rope is pulled at a rate of 3 m/sec.

(a) Draw and label a figure that illustrates this situation.

(b) How fast is the boat approaching the dock when it is 9 m from the dock?

IV. State the Mean Value Theorem, including its hypotheses.
(8)

(b) For the function $\sqrt{x}$ on the interval $[0, 4]$, find a number c that satisfies the conclusion of the Mean Value Theorem.

V. (4) Use the Mean Value Theorem to verify the following fact: Suppose that $f\colon [a,b] \to \mathbb{R}$ is continuous on $[a,b]$ and differentiable on (a,b). If $f'(x) > 0$ for $a < x < b$, then $f(a) < f(b)$.

VI. (8) A certain differentiable function f has domain all real numbers except $x = -2$, and has the following properties:

(a) $f(-3) = -1$, $f(1) = 1$, $f(3) = 2$

(b) $f'(-3) = 0$, $f'(1) = 0$.

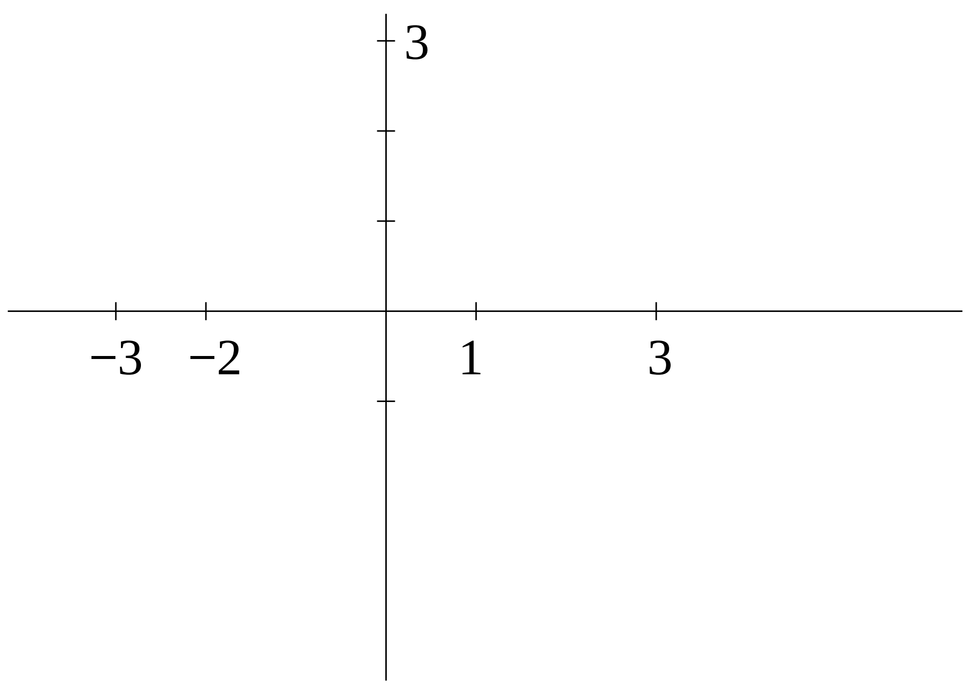
(c) $\lim_{x \to -2} f(x) = -\infty$.

(d) $f''(x) < 0$ for $x < -2$, for $-2 < x < 1$, and for $3 < x$.

(e) $f''(x) > 0$ for $1 < x < 3$.

(f) $\lim_{x \to \infty} f(x) = 3$.

Sketch a possible graph of f, using all of the above information.



VII. (10) Let $f\colon [a,b] \to \mathbb{R}$ be a function which is continuous on $[a,b]$ and differentiable on (a,b). Label each of the following statements either T for true or F for false.

_______ If $f'(x) < 0$ for $a < x < b$, then $f(a) < f(b)$.

_______ If a and b are roots of f, then there must exist a number c between a and b for which $f'(c) = 0$.

_______ The Mean Value Theorem can be deduced from Rolle's Theorem.

_______ The Mean Value Theorem is a special case of Rolle's Theorem.

_______ If $f(a) < f(b)$, then there must exist a number c between a and b for which $f'(c) > 0$.

_______ There must exist a number c in the interval $[a,b]$ such that $f(c) \geq f(x)$ for all x in $[a,b]$.

_______ If f were not continuous on $[a,b]$, then it could still have an absolute maximum or absolute minimum value on $[a,b]$, but not both.

_______ If c is an interior point of the interval and $f'(c) = 0$, then f must have either a local maximum or a local minimum (or both) at c.

_______ If $f''(x)$ changes sign at c, then c is an inflection point of f.

_______ If $f(x)$ equals the mass of the portion of a metal rod between 0 and x, then $f'(x)$ is the density function of the rod.