Mathematics 1823-030
Examination III Form B
November 23, 2009

Name (please print) ______________________

Student Number ______________

(1) **Discussion Section** (circle day and time):

Th 9:00 Th 1:30 Th 3:00 F 8:30 F 9:30 F 2:30

I. (a) Find the differential of $\sqrt{x}$.
(7)

(b) Use linear approximation to estimate $\sqrt{3.996}$.

II. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 2 m higher than the bow of the boat. The rope is pulled at a rate of 2 m/sec.
(8)

(a) Draw and label a figure that illustrates this situation.

(b) How fast is the boat approaching the dock when it is 12 m from the dock?

III. In this problem, $f(x) = \frac{2}{x} + \frac{1}{x^2}$, which can also be written as $\frac{2}{x^2}\left(x + \frac{1}{2}\right)$. The first and second derivatives
(10) of f are $f'(x) = -\frac{2}{x^2} - \frac{2}{x^3}$ and $f''(x) = \frac{4}{x^3} + \frac{6}{x^4}$ (do not check these)

1. Find the root or roots of f.

2. Find $\lim\limits_{x\to\infty} f(x)$.

3. Find $\lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to 0^-} f(x)$.

4. Find the critical number or critical numbers of f.

5. Find the inflection point or inflection points of f.

IV. State the Mean Value Theorem, including its hypotheses.
(4)

V. (8) (a) For the function $\sqrt{x}$ on the interval $[0, 9]$, find a number c that satisfies the conclusion of the Mean Value Theorem.

(b) Use the Mean Value Theorem to verify the following fact: Suppose that $f\colon [a, b] \to \mathbb{R}$ is continuous on $[a, b]$ and differentiable on (a, b). If $f'(x) < 0$ for $a < x < b$, then $f(a) > f(b)$.

VI. (8) A certain differentiable function f has domain all nonzero real numbers, and has the following properties:

(a) $f(-2) = 1$, $f(3) = 4$, $f(5) = 2$

(b) $f'(-2) = 0$, $f'(3) = 0$.

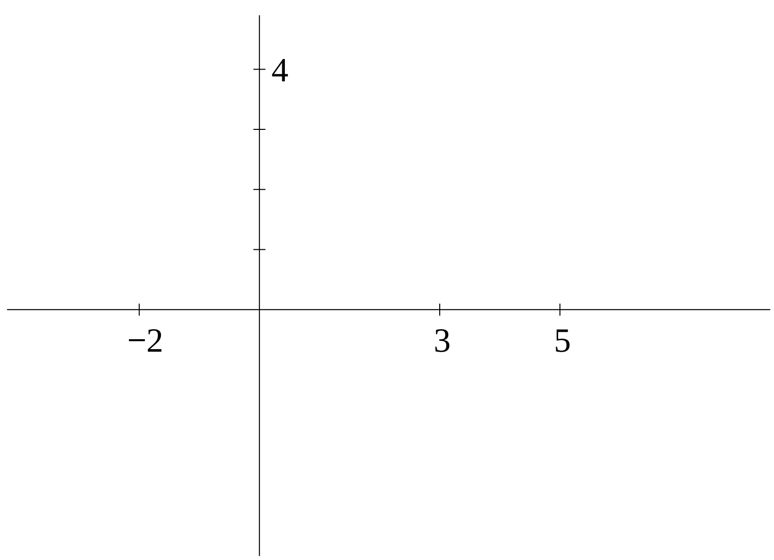
(c) $\lim_{x\to 0} f(x) = -\infty$.

(d) $f''(x) < 0$ for $-2 < x < 0$ and for $0 < x < 5$.

(e) $f''(x) > 0$ for $x < -2$ and $5 < x$.

(f) $\lim_{x\to\infty} f(x) = 1$.

Sketch a possible graph of f, using all of the above information.



VII. Let $f\colon [a,b] \to \mathbb{R}$ be a function which is continuous on $[a,b]$ and differentiable on (a,b). Label each of the following statements either T for true or F for false.
(10)

_______ If a and b are roots of f, then there must exist a number c between a and b for which $f'(c) = 0$.

_______ If $f'(x) > 0$ for $a < x < b$, then $f(a) < f(b)$.

_______ The Mean Value Theorem is a special case of Rolle's Theorem.

_______ The Mean Value Theorem can be deduced from Rolle's Theorem.

_______ If $f(a) < f(b)$, then there must exist a number c between a and b for which $f'(c) > 0$.

_______ If $f''(x)$ changes sign at c, then c is an inflection point of f.

_______ There must exist a number c in the interval $[a,b]$ such that $f(c) \geq f(x)$ for all x in $[a,b]$.

_______ If f were not continuous on $[a,b]$, then it could still have an absolute maximum or absolute minimum value on $[a,b]$, but not both.

_______ If c is an interior point of the interval and $f'(c) = 0$, then f must have either a local maximum or a local minimum (or both) at c.

_______ If $f(x)$ equals the mass of the portion of a metal rod between 0 and x, then $f'(x)$ is the density function of the rod.