Math 2423 homework

7. (due 8/31) Expand each of the following summations as a sum of individual terms, then make reasonable simplifications where possible.

   (a) $\sum_{n=1}^{5} \sin(nx)$

   (b) $\sum_{x=1}^{5} \sin(nx)$

   (c) $\sum_{t=-3}^{3} f(1-t)$

   (d) $\sum_{j=1}^{m} g(x_j^*)\Delta x_j$

   (e) $\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

8. (due 8/31) Write each of the following in summation notation:

   (a) $a_0 + a_1 + a_4 + a_9 + a_{16} + a_{25}$

   (b) $s(t_0^*)\Delta t_0 + s(t_1^*)\Delta t_1 + \cdots + s(t_n^*)\Delta t_n$

9. (due 8/31) Obtain the formula $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ as follows.

   (a) Write $n^4 = \sum_{i=1}^{n} i^4 - (i-1)^4$ (be sure you understand why this is true).

   (b) Simplify $i^4 - (i-1)^4$.

   (c) Use the simplification to break up $\sum_{i=1}^{n} i^4 - (i-1)^4$ as a sum of terms which may involve $\sum_{i=1}^{n} i$, $\sum_{i=1}^{n} i^2$, and $\sum_{i=1}^{n} i^3$.

   (d) Substitute in the already-established formulas $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$, then solve for $\sum_{i=1}^{n} i^3$ and simplify to obtain the formula $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.